LECTURE:3A Mechanical and electrical analogues of forced vibration

The EMF equation of series LCR ckt with ac source:

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = Ve^{j\omega t}$$

This equation is similar in form to eq of forced vibration.

m is analogous to L, k to R,1/s to C, F to V, x to charge q, dx/dt to current.

L: inductance

R: resisrtance

C: capacitance

V: voltage/

In steady state charge: $q = \frac{Ve^{j\omega t}}{w|z_m|}$, written from analogy.

In steady state current: $i = \frac{dq}{dt} = \frac{Ve^{j\omega t}}{|z_m|}$

$$|z_m| = \left[\left(\frac{L.\frac{1}{LC}}{w} - Lw\right)^2 + \frac{R^2}{L^2}L^2\right]^{\frac{1}{2}} = \left[\left(\frac{1}{wC} - Lw\right)^2 + R^2\right]^{\frac{1}{2}} \quad w_0^2 = \frac{s}{m} = \frac{1}{LC}, \quad 2b = \frac{k}{m} = \frac{R}{L}$$

WRITTEN FROM ANALOGY OF

$$|z_m| = \left[\left(\frac{mw_0^2}{w} - mw\right)^2 + 4b^2m^2\right]^{1/2}$$

IMPEDENCE: $|z_m| = \left[\left(\frac{1}{wC} - Lw\right)^2 + R^2\right]^{\frac{1}{2}}$

COMPLEX IMPEDENCE: $Z = R + J\left(wL - \frac{1}{wC}\right)$

REACTANCE: $X = wL - \frac{1}{wC}$

Current lags the driving voltage by $\emptyset$