DIFFRACTION II
SINGLE SLIT FRAUNHOFER DIFFRACTION

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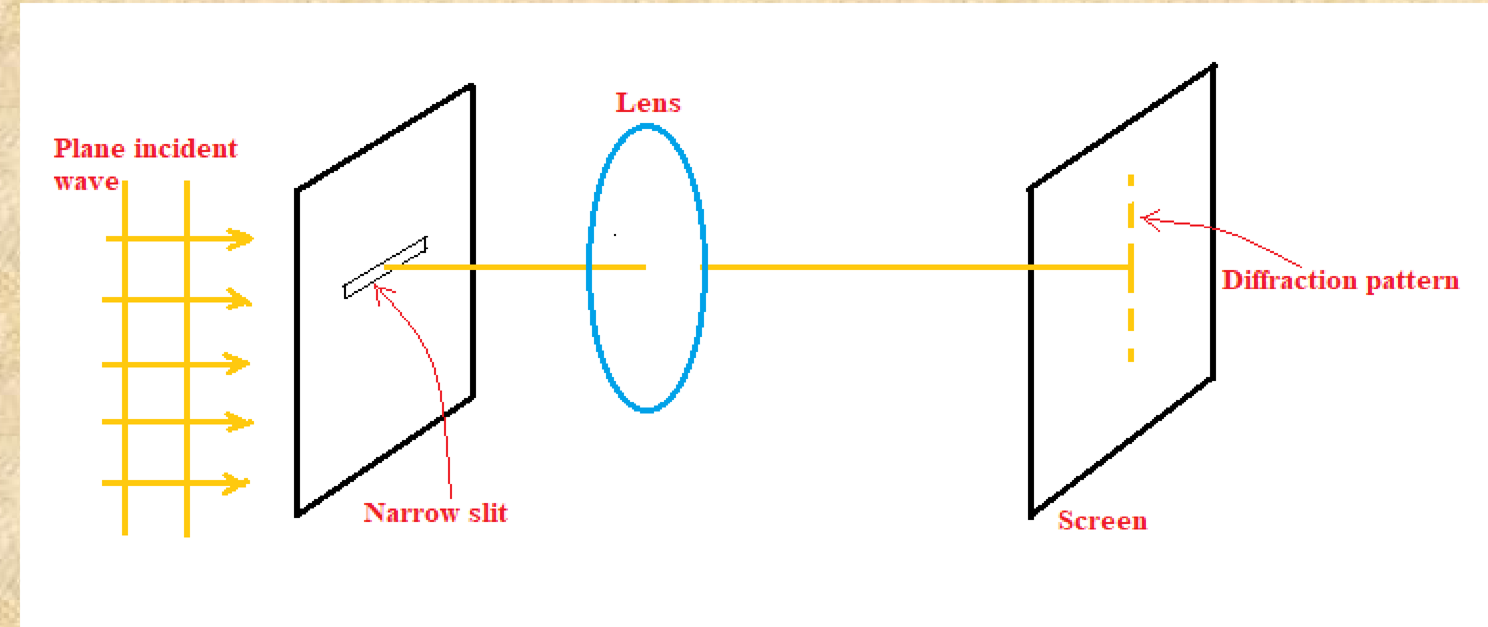
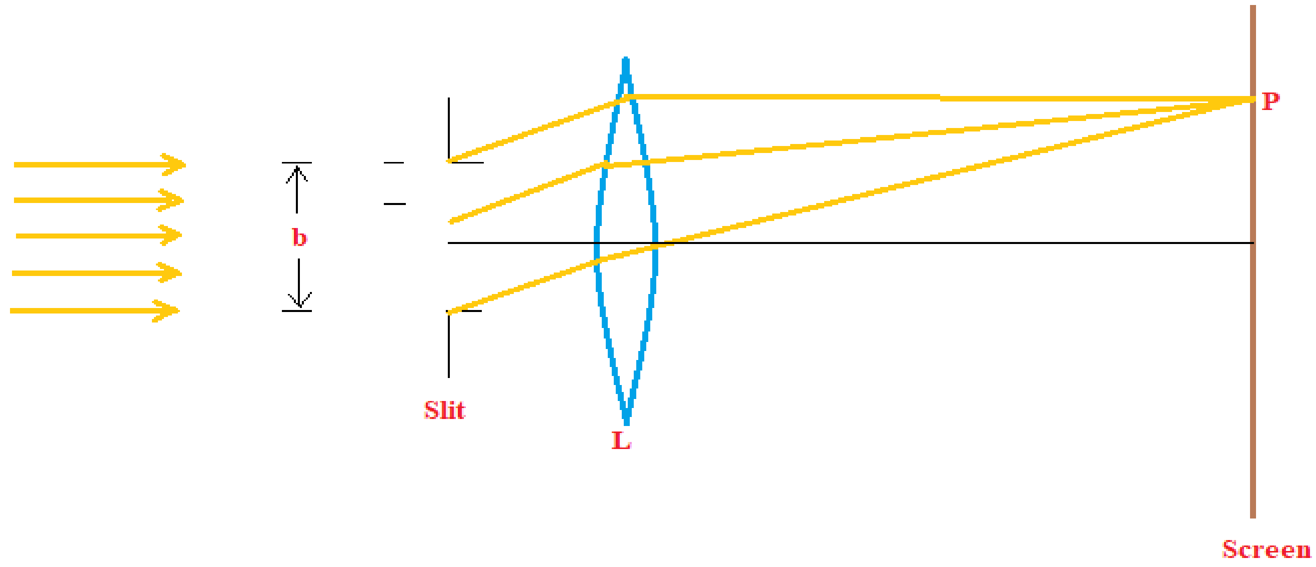
SINGLE SLIT DIFFRACTION PATTERN

This is a narrow slit on an opaque plane. Plane waves are incident on the slit. Let the width of the slit be b along the y axis. The diffraction pattern that we will be seen on the screen placed at the focal plane of the lens.



Let us take a section of the slit ignoring the larger dimension of the slit.
According to Huygens-Fresnel Principle each point on the slit will acts as
a source of secondary wavelet. Let two consecutive secondary source are separated by a distance y. The slit consists of infinite number of sources or we may say a continuous distribution of sources. The path difference between the two waves emanating from two consecutive point sources would be

$$y\sin\theta$$



Where θ is the angle at which the waves diffracted from the source.
Corresponding phase difference

$$\delta = \frac{2\pi}{\lambda} y \sin\theta$$

$$= ky$$

Where $k = \frac{2\pi \sin\theta}{\lambda}$

Let the origin is at the centre and the aperture length varies from b/2 to –b/2.
Let us consider a small element dy at the centre.
The amplitude of the secondary waves that comes from the element is

$$d\tilde{E} = \widetilde{E_0}\ dy$$

Integrating from –b/2 to b/2

$$e^{i\delta} d\tilde{E} = e^{i\delta} \widetilde{E_0} dy$$

$$\tilde{E} = \widetilde{E_0} \int_{-b/2}^{b/2} e^{iky} dy$$

$$= \widetilde{E_0} \left[\frac{e^{ikb/2} - e^{-ikb/2}}{ik}\right]$$

$$= \widetilde{E_0} \frac{2i \sin\frac{kb}{2}}{ik}$$

$$= \widetilde{E_0}\, b \frac{\left[\sin \frac{b\pi \sin\theta}{\lambda}\right]}{\frac{b\pi \sin\theta}{\lambda}}$$

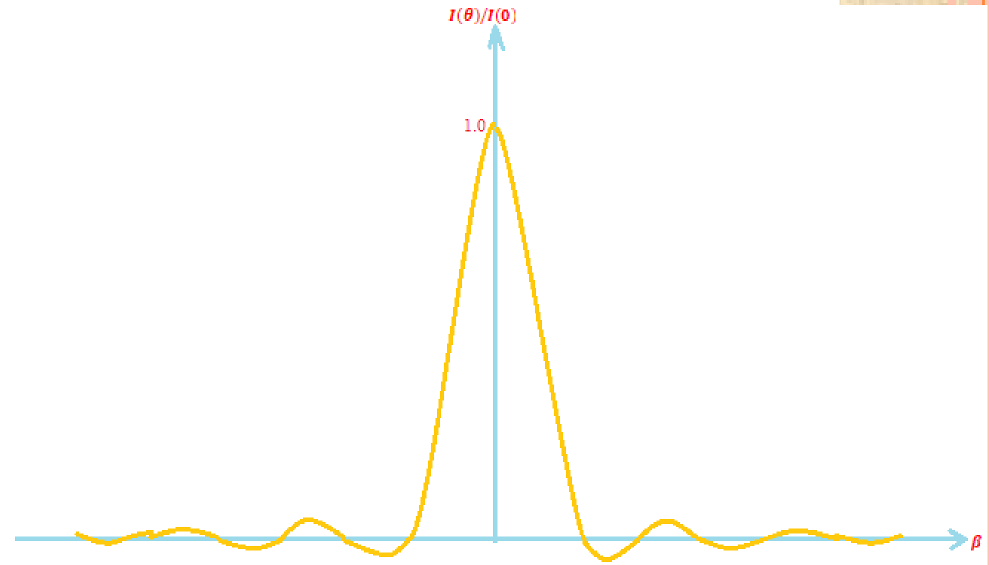
$$= \widetilde{E_0}\, b \frac{\sin\beta}{\beta}$$

Where $\beta = \frac{\pi b \sin\theta}{\lambda}$

Intensity $I(\theta) = \frac{1}{2} \tilde{E}\tilde{E}^*$

$$= I(0)\left(\frac{\sin\beta}{\beta}\right)^2$$

$$= I(0)\,\mathrm{sinc}^2\beta$$



When θ = 0, $\frac{sin\beta}{\beta}$ = 1 and $I(\theta)$=$I(0)$, that corresponds to principal Maxima. As θ = 0 here

Thus the intensity resulting from an idealized coherent line source in a Fraunhofer approximation is

$$I(\theta) = I(0)sinc^2\beta$$

When b≫ λ , the intensity drops very rapidly as θ deviates from zero.

When λ≫ b, β is small sinß = ß gives $I(\theta) = I(0)$

If θ≪ 1 , $I(\theta) = I(0)sinc^2(\frac{\pi b\theta}{\lambda})$

$\theta = 0$, gives $I(\theta)$=$I(0)$

For $\beta = \pm m\pi$

$I(\theta)$= 0

$\beta = \frac{\pi b\theta}{\lambda}$

$\theta = \pm m\frac{\lambda}{b}$

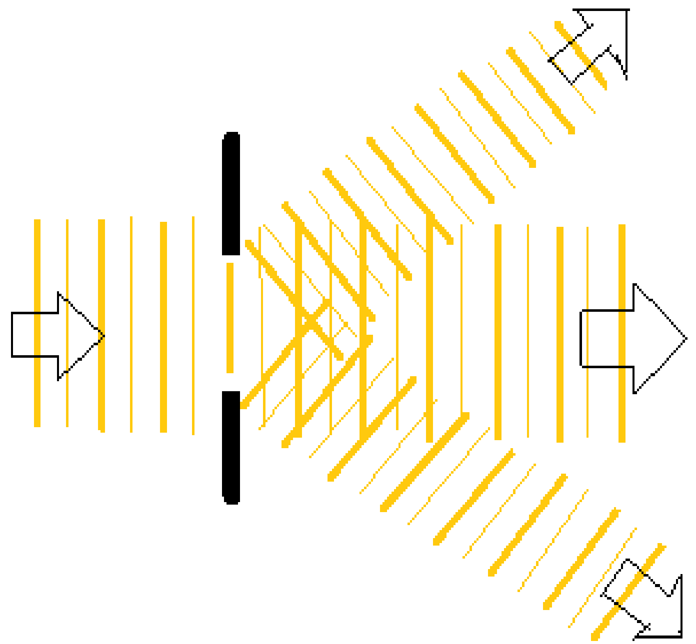
Smaller the value of b

Larger be the width of the

Central maximum

$$I(\theta) = I(0)\, sinc^2(\beta_x) sinc^2(\beta_y)$$

Where $\beta_x = \frac{\pi b_x sin\theta_x}{\lambda}$ and $\beta_y = \frac{\pi b_y sin\theta_y}{\lambda}$



POSITION OF MAXIMA AND MINIMA

We have $I(\theta) = I(0)\frac{sin\beta}{\beta} = I(0)\frac{\sin(\frac{\pi b\theta}{\lambda})}{\frac{\pi b\theta}{\lambda}}$

$\frac{\pi b\theta}{\lambda} = \pm\frac{m\lambda}{b}$ where m = 1, 2, 3, ...

$$\theta = \pm\frac{m\lambda}{b}$$

First minimum $\theta = \pm\frac{\lambda}{b}$

In order to get the position of maxima

$$\frac{dI}{d\beta} = 0$$

$$\frac{dI}{d\beta} = I(0)[\frac{2sin\beta cos\beta}{\beta^2} - \frac{2sin^3\beta}{\beta^3}] = 0$$

$$sin\beta(\beta - tan\beta) = 0$$

The conditions for maxima are the roots of the transcendental equation

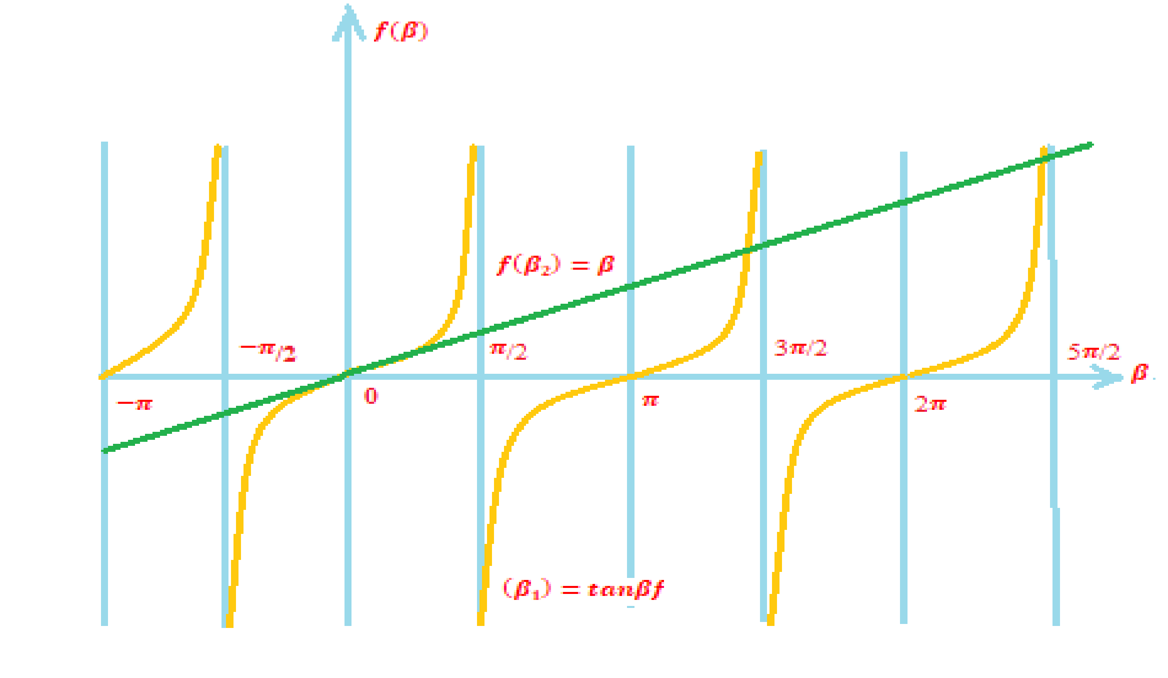
$tan\beta = \beta$

The intersections of the curves $y = \beta$ and $y = tan\beta$ occur at $\beta = \pm 1.43\pi$, $\pm 2.46\pi$, $\pm 3.47\pi$, ...

The intensity of the first minimum

$$I(1) = I(0)\left(\frac{sin1.43\pi}{1.43\pi}\right)^2 = 0.0496I(0)$$

Thus the intensity of the first mininmum is about 4.96% of central maximum. Similarly the intensity of the 2nd, 3rd, ... maximas are about 1.68% , 0.83%, ... of the central maximum respectively.



INTENSITY DISTRIBUTION