FABRY-PEROT INTERFEROMETER II

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We have the expression of intensity from the last lecture

$$I = \frac{T^2}{(1-R)^2[1+\frac{4R}{(1-R)^2}sin^2\frac{\delta}{2}]}$$

CONDITION FOR MAXIMA

For maximum intensity of the of the fringe

$$sin^2\frac{\delta}{2} = 0 \;=>\; \frac{\delta}{2} = n\pi$$

$\delta = 2n\pi$, n = 0, 1, 2, 3, ...

Maximum intensity

$$I_{max} = \frac{T^2}{(1-R)^2}$$

CONDITION FOR MINIMA

The intensity is minimum when

$$sin^2\frac{\delta}{2} = 1$$

$$\frac{\delta}{2} = (2n+1)\frac{\pi}{2} => \delta = (2n+1)\pi, n = 0, 1, 2, 3, \ldots$$

Minimum intensity, $$I_{mim} = \frac{T^2}{(1-R)^2[1+\frac{4R}{(1-R)^2}]}$$

Hence,

$$I_{min} = \frac{T^2}{(1+R)^2}$$

We know A+ R + T = 1

When there is no absorption at the reflecting surface, i.e. A = 0, we can write R + T = 1

$$I_{max} = 1$$

And

$$I_{min} = \frac{(1-R)^2}{(1+R)^2}$$

$$\frac{I_{max}}{I_{mim}} = \frac{(1+R)^2}{(1-R)^2}$$

Also $$\frac{I}{I_{max}} = \frac{1}{1+\frac{4R}{(1-R)^2}\sin^2\frac{\delta}{2}}$$

$$= \frac{1}{(1+F\sin^2\frac{\delta}{2})}$$

Where $F = \frac{4R}{(1-R)^2}$

CIRCULAR SHAPE OF FRINGES

In terms of path difference,

For maxima: $2\,t\cos\theta = n\,\lambda$

For minima: $2\,t\cos\theta = (2n-1)\frac{\lambda}{2}$

For constant t, for a particular n and λ, θ is a constant.

VISIBILITY OF FRINGES

The visibility of the fringe is defined as

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

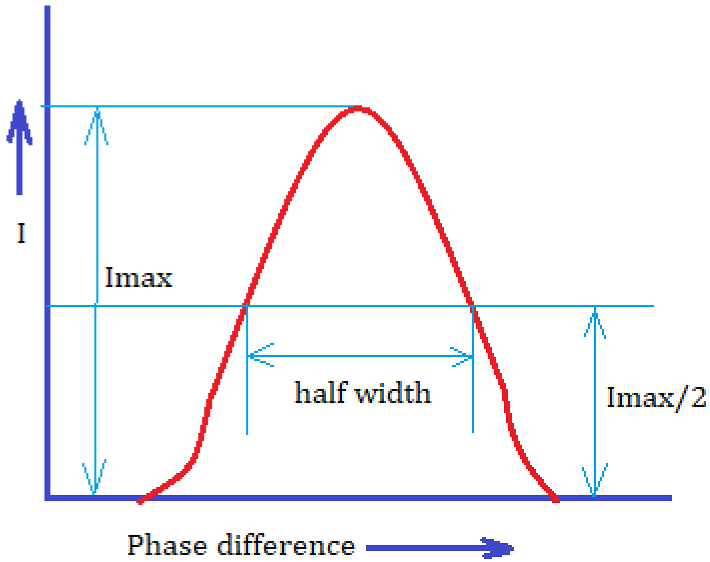
$$= \frac{2R}{1+R^2}$$

Thus, the visibility of the fringes depends only on the reflection coefficient of the silver coating and is independent of its transmission coefficient.

SHARPNESS OF FRINGES – HALF FRINGE WIDTH

The half fringe width of a fringe is the width of fringe in terms of phase difference between the two points on either sides of maxima, where the intensity is half of its maximum value.

$$\frac{I}{I_{max}} = \frac{1}{1+F\,sin^2\frac{\delta}{2}}$$



At half fringe width

$$\frac{I}{I_{max}} = \frac{1}{2}$$

$$\frac{1}{1+F\,sin^2\frac{\delta}{2}} = \frac{1}{2}$$

$$1 + F sin^2\frac{\delta}{2} = 2$$

$$F sin^2\frac{\delta}{2} = 1$$

$$\sin\frac{\delta}{2} = \frac{1}{\sqrt{F}} = \frac{1}{\sqrt{[\frac{4R}{(1-R)^2}]}} = \frac{(1-R)}{2\sqrt{R}}$$

$$\delta = 2\sin^{-1}(\frac{1-R}{2\sqrt{R}})$$

For small value of δ

$$\sin\frac{\delta}{2} = \frac{\delta}{2}$$

$$\delta = \frac{1-R}{\sqrt{R}}$$

Thus, smaller is the half fringe width, sharper is the bright fringe.
Now we calculate the values of δ for various values of R and corresponding value of $^I/_{I_{max}}$.

For R = 0.25

$\delta = 1.5$

$\frac{I}{I_{max}} = 0.9997$

For R = 0.50

$$\delta = 0.707$$

$$\frac{I}{I_{max}} = 0.9997$$

For R = 0.75

$$\delta = 0.2887$$

$$\frac{I}{I_{max}} = 0.9997$$

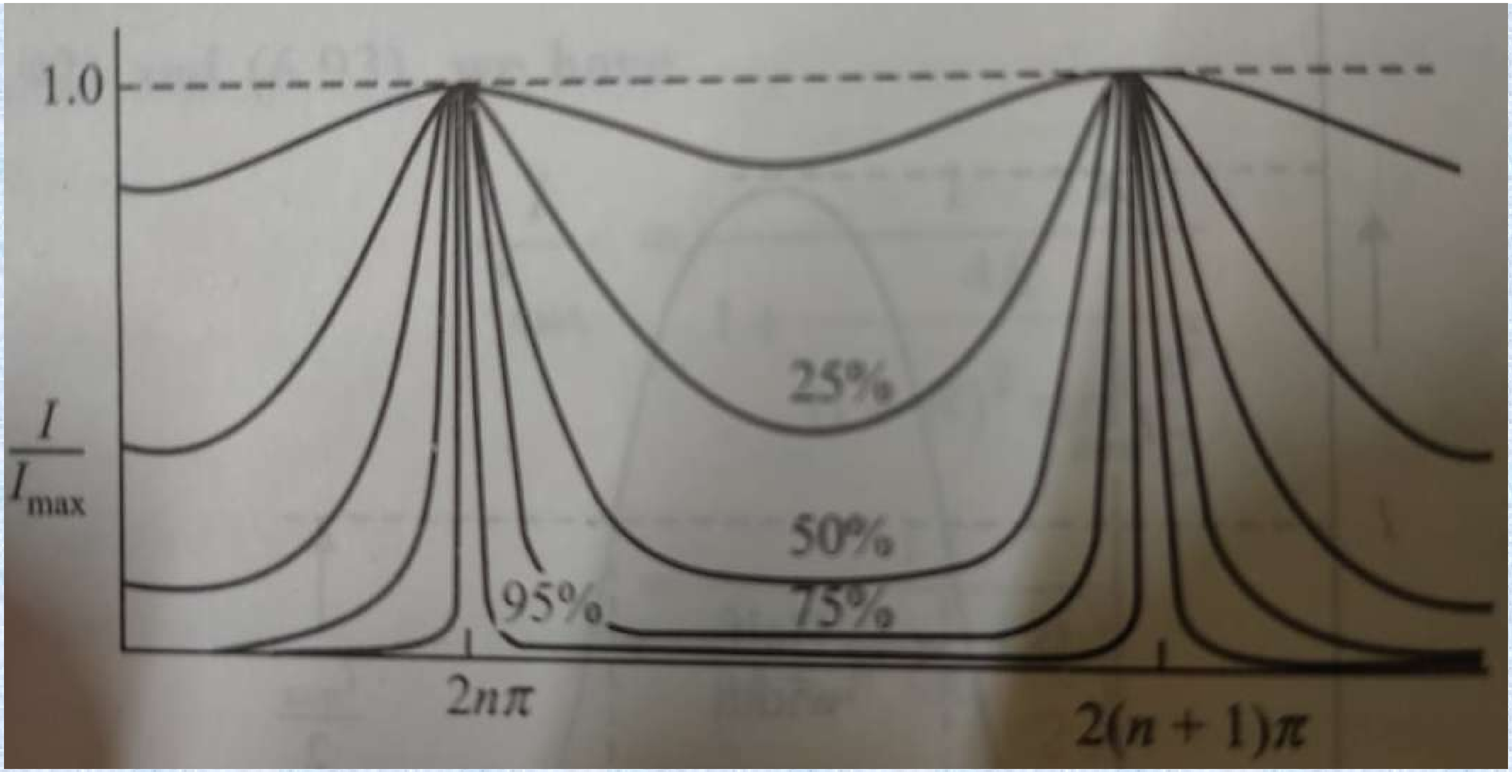
For R = 0.95

$$\delta = 0.0513$$

$$\frac{I}{I_{max}} = 0.9997$$

It is obvious from the above calculation that higher the value of R, lower the value of δ, thus sharper is the maxima. But the value of I/I_{max} remains same.

So, if we plot the variation of I/I_{max} with phase difference δ for different values of R, it will look like this.



DETERMINATION OF WAVELENGTH

The interferometer is adjusted so that circular fringes are obtained. Let n be the order of fringe at the centre, so that

$$2\,t = n\,\lambda$$

Now, the movable plate is moved from a known distance, say from x_1 *to* x_2 and let N be the number of fringes that disappear at the centre.

Thus

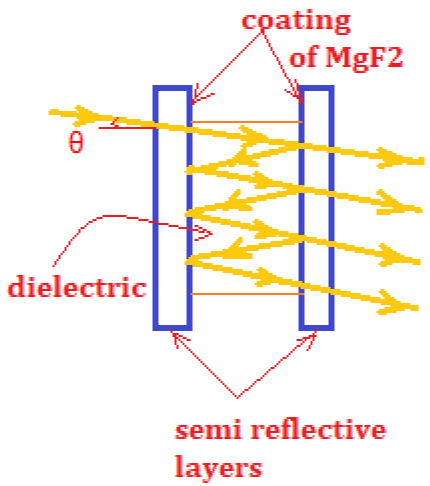
$$N\frac{\lambda}{2} = x_2 - x_1$$

$$\lambda = \frac{2(x_2 - x_1)}{N}$$

APPLICATION OF FABRY-PEROT INTERFEROMETER

With increasing use of lasers in various fields, the phenomenon of interference is widely used in various measuring and testing equipments.

INTERFERENCE FILTERS: This provides the selection of narrowband of light wavelengths $\leq 100\,\mathring{A}$ out of complete spectrum.



For chromatic light, transparent beam shall interfere constructively i.e. the path difference should be integral multiple of wavelength.

$$2\mu t = n\lambda$$

$$\lambda = \frac{2\mu t}{n}$$

If the thickness of the film is so adjusted that $\mu t = \lambda_0$, then the transmitted or the filtered wave lengths shall be $2\lambda_0, \lambda_0, \frac{2\lambda_0}{3}, \ldots$

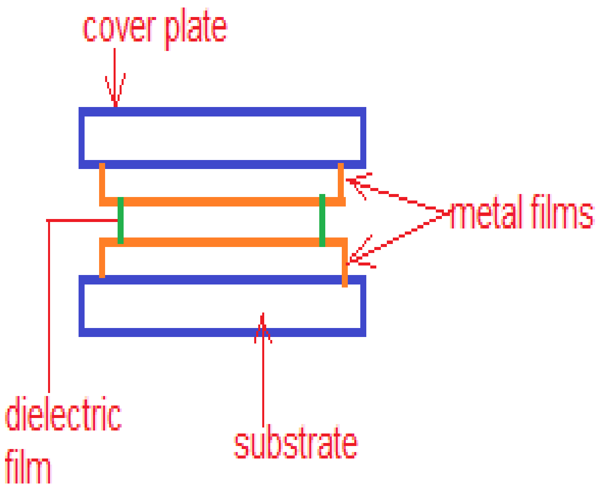
If t is large a large number of maxima will be observed in the visible range; about 23,000 maxima are observed if t = 1 cm.

If $\mu = 1.5$ and $t = 6 \times 10^{-5}$ cm, there are only two maxima in the visible region.

$\lambda = 6000\dot{A}$ for n = 3 and

$\lambda = 4500\ \dot{A}$ for n = 4.

Modern vacuum deposition technique is used to obtain interference filters.



A thin metallic film, usually aluminium or silver is deposited on a substrate (glass). Then a thin layer of dielectric material such as cryolite ($3NaF, AlF_3$) is deposited over this. This structure is This structure is again covered by another metallic film and another glass plate is placed over it. Thus a Fabry-Perot structure is formed between two glass plates. The sharpness of the transmitted spectrum is determine by the resolving power of the Fabry-Perot structure and hence by the reflectivity of the surface.

The larger the reflectivity, the narrower is the transmitted spectrum.

ANTIREFLECTION COATING:

It is now common practice to apply antireflection coatings to the elements of optical instruments.

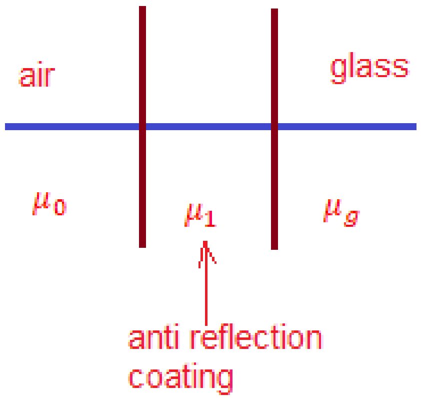
For air-glass boundary, the reflectivity R = 0.04 i.e. 4%.

So, when light passing from air to glass system, a part is lost due to reflection at each lens surface.

This loss may be eliminated by applying a suitable coating of suitable thickness to the lens surface. The light beams reflected from the two surfaces of the coating, then mutually cancel.

For single coating,

$$R_1 = \frac{(\mu_0\mu_g - \mu_1^2)^2}{(\mu_0\mu_g + \mu_1^2)^2}$$



$$R_1 = 0, \mu_1^2 = \mu_0\mu_g$$

Generally, for normal incidence

$$2\mu_1 t = \frac{\lambda}{2}$$

$$t = \frac{\lambda}{4}\mu_1$$

The value of t is chosen so that the yellow green portion of the visible spectrum do not allow to be reflected.

A single $\frac{\lambda_0}{4}$ layer of MgF2 will reduce the reflectance of glass from about 4% to a bit more than 1%, over visible spectrum.
At wavelengths on either side of the central yellow green region, R increases and the lens surface will appear blue red in reflected light.
For a double layer, quarter wavelength antireflection coating

$$\mu = \mu_1 \mu_2.$$

That gives

$$R_2 = \frac{(\mu_2^2\mu_0 - \mu_g\mu_1^2)^2}{(\mu_2^2\mu_0 + \mu_g\mu_1^2)^2}$$

For R_2 to be exactly zero,

$$\mu_2 = \mu_1\sqrt{\frac{\mu_g}{\mu_0}}$$

This kind of film is referred to as a double quarter, single minimum coating.
The common practice to designate the system as gHLa => glass-high index-low index-air.

Problem1: White light is incident normally on a Fabry-Perot interferometer with a plate separator of 4×10^{-4} cm. Calculate the wavelengths for which there are interference maxima in the transmitted beam in the range $4000\dot{A}$ to $5000\ \dot{A}$

Problem 2: Find minimum thickness required of a layer of cryolite(μ=1.35) in an interference filter designed to isolate light of wavelength 594 nm. How the peak transmittance change if filter is tilted by 10 degree?

Problem 3: Two thick layers are deposited on an ophthalmic glass (μ=1.52) to reduce reflection loss. The 1st layer is MgF_2, what is the refractive index of the 2nd layer?

Problem 4: A glass microscope lens having an index of 1.55 is to be coated with MgF_2 film to increase the transmission of normally incident yellow light(λ_0=550 nm). What minimum thickness should be deposited on the lens?