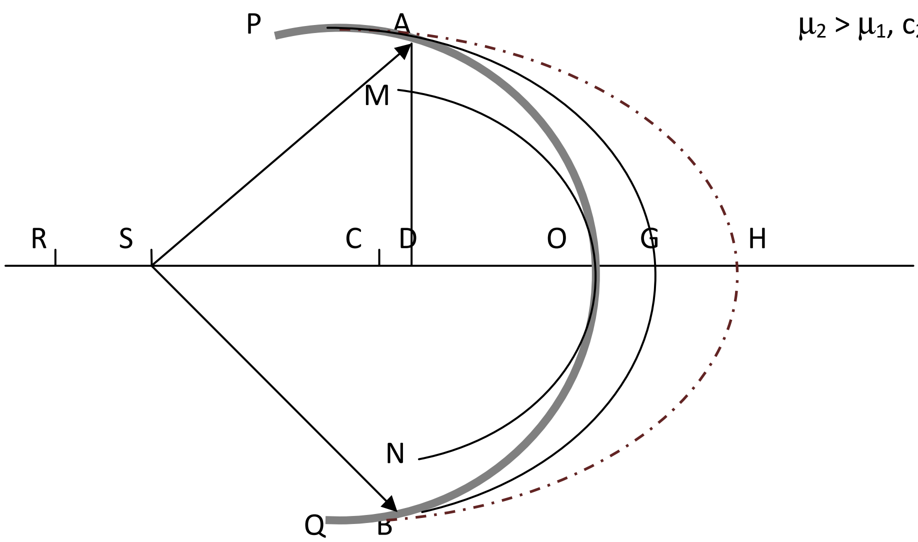
# APPLICATION OF HUYGEN'S CONSTRUCTION TO FIND EQUATION ODF REFRACTION AT A CURVED SURFACE

## 1. CONCAVE SURFACE

**a) Source or object at the rarer medium.**

($\mu_1$, $c_1$) ($\mu_2$, $c_2$)




$\mu_2 > \mu_1$, $c_2 < c_1$

Fig. 3.1: Demonstration of refraction at a concave surface from rarer to denser medium.

PQ $\rightarrow$ concave refracting surface separating two media of refractive indices $\mu_1$ and $\mu_2$

C $\rightarrow$ centre of curvature of MN

OC $\rightarrow$ radius of curvature = -r

$c_1 \rightarrow$ velocity of light in medium $\mu_1$

$c_2 \rightarrow$ velocity of light in medium $\mu_2$

S $\rightarrow$ object or source

MON $\rightarrow$ Incident wave front

S $\rightarrow$ centre of the spherical wave front MN

OS $\rightarrow$ object distance = -u

AHB $\rightarrow$ wave front after time 't' in absence of second medium

S $\rightarrow$ centre of the spherical wave front AHB

OH $\rightarrow$ distance travelled by the wave front in time 't' in absence of second medium = $c_1t$

AGB $\rightarrow$ Refracted wave front after time 't'

R $\rightarrow$ centre of the spherical wave front AGB. R is the real image of S

OR $\rightarrow$ Image distance = -v

OG $\rightarrow$ distance travelled by the wave front in time 't' in $\mu_2$ = $c_2t$

AD $\rightarrow$ perpendicular dropped from A on the principal axis

<u>From the geometry of the fig. 3.2</u>

Considering arc AOB with centre C

$$(2OC - OD).OD = AD^2$$

As AOB has a large radius of curvature and small aperture: OD<<OC . Hence $2OC - OD \approx 2OC$

$\therefore$ $2OC.OD = AD^2$ ……………………………………(3.1)

Similarly Considering arc AHB with centre S

$$2HS.HD = AD^2$$

For small aperture H and O are almost overlapping

$\therefore$ $2OS.HD = AD^2$ ……………………………………(3.2)

In the same way considering arc AGB with centre with same approximations

$2OR.GD = AD^2$ …………………………………………(3.3)

Now $\frac{c_1t}{c_2t} = \frac{HO}{GO} = \frac{HD-OD}{GD-OD} = \frac{\frac{AD^2}{2}\left(\frac{1}{OS}-\frac{1}{OC}\right)}{\frac{AD^2}{2}\left(\frac{1}{OR}-\frac{1}{OC}\right)} = \frac{\frac{1}{OS}-\frac{1}{OC}}{\frac{1}{OR}-\frac{1}{OC}} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$ ……. From (3.1) (3.2) (3.3)

Or $\frac{c_1}{c_2} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\frac{c_0}{c_2}}{\frac{c_0}{c_1}} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\mu_2}{\mu_1} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\mu_2\left(-\frac{1}{v}+\frac{1}{r}\right) = \mu_1\left(-\frac{1}{u}+\frac{1}{r}\right)$

Or $-\frac{\mu_2}{v} + \frac{\mu_2}{r} = -\frac{\mu_1}{u} + \frac{\mu_1}{r}$

Or $\frac{\mu_2 - \mu_1}{r} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$

Or $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}}$ ……………………………………………….(3.4)

Equation (3.4) gives the equation of refraction for convex surface.

**b) Source or object at the denser medium.**

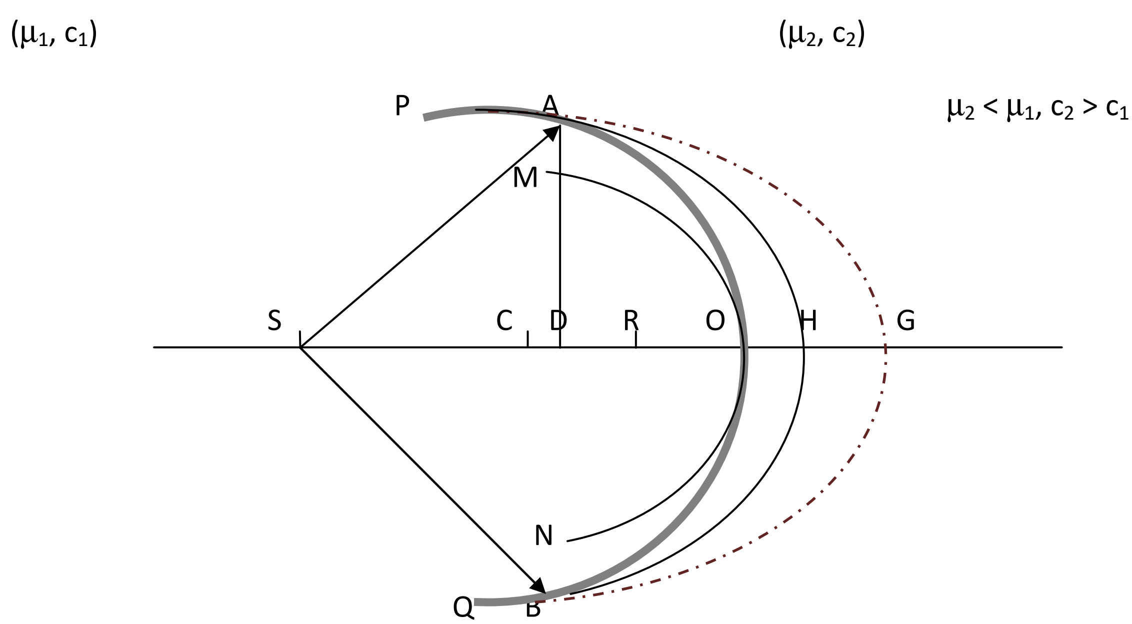



Fig. 3.2: Demonstration of refraction at a concave surface from denser to rarer medium.

**Remaining part is exactly as the previous one. We get the same equation as (3.4)**

**2. CONVEX SURFACE**

**a) Source or object at the rarer medium.**

$(\mu_1, c_1)$ $(\mu_2, c_2)$

$\mu_2 > \mu_1,\ c_2 < c_1$

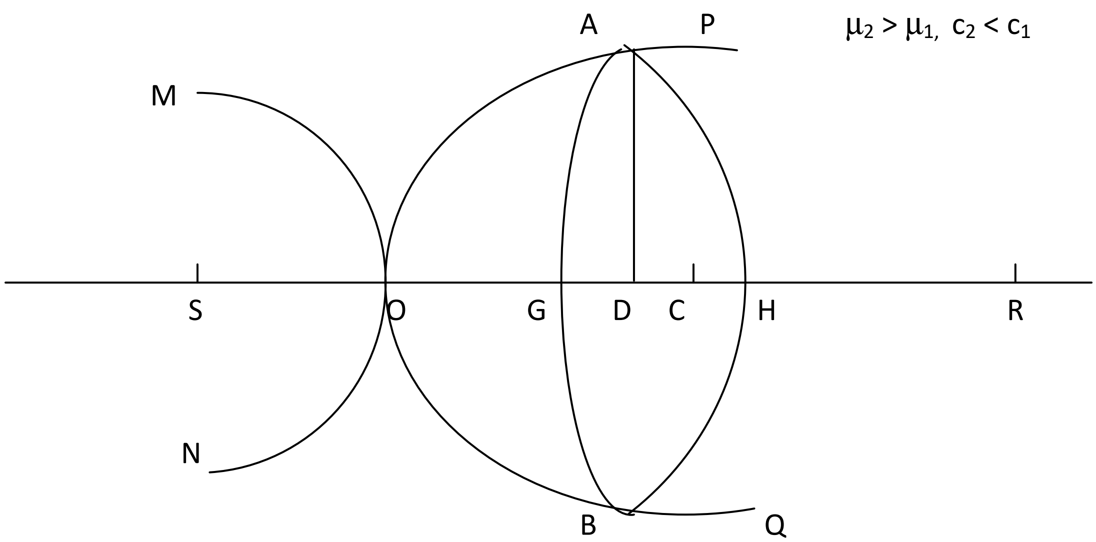



Fig. 3.3: Demonstration of refraction at a convex surface from rarer to denser medium.

PQ → concave refracting surface separating two media of refractive indices $\mu_1$ and $\mu_2$

C → centre of curvature of MN

OC → radius of curvature = r

$c_1$ → velocity of light in medium $\mu_1$

$c_2$ → velocity of light in medium $\mu_2$

S → object or source

MON → Incident wave front

S → centre of the spherical wave front MN

OS → object distance = -u

AHB → wave front after time ‘t’ in absence of second medium

S → centre of the spherical wave front AHB

OH → distance travelled by the wave front in time ‘t’ in absence of second medium = $c_1t$

AGB $\rightarrow$ Refracted wave front after time 't'

R $\rightarrow$ centre of the spherical wave front AGB. R is the real image of S

OR $\rightarrow$ Image distance = v

OG $\rightarrow$ distance travelled by the wave front in time 't' in $\mu_2 = c_2t$

AD $\rightarrow$ perpendicular dropped from A on the principal axis

<u>From the geometry of the fig. 3.2</u>

Considering arc AOB with centre C

$$(2OC - OD).OD = AD^2$$

As AOB has a large radius of curvature and small aperture: OD<<OC . Hence $2OC - OD \approx 2OC$

$\therefore$ $2OC.OD = AD^2$ ……………………………………(3.1)

Similarly Considering arc AHB with centre S

$$2HS.HD = AD^2$$

For small aperture H and O are almost overlapping

$\therefore$ $2OS.HD = AD^2$ ……………………………………(3.2)

In the same way considering arc AGB with centre with same approximations

$2OR.GD = AD^2$ …………………………………………(3.3)

Now $\frac{c_1t}{c_2t} = \frac{HO}{GO} = \frac{HD+OD}{OD-GD} = \frac{\frac{AD^2}{2}\left(\frac{1}{OS}+\frac{1}{OC}\right)}{\frac{AD^2}{2}\left(\frac{1}{OC}-\frac{1}{OR}\right)} = \frac{\frac{1}{OS}+\frac{1}{OC}}{\frac{1}{OC}-\frac{1}{OR}} = \frac{-\frac{1}{u}+\frac{1}{r}}{\frac{1}{r}-\frac{1}{v}}$ ……. From (3.1) (3.2) (3.3)

Or $\frac{c_1}{c_2} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\frac{c_0}{c_2}}{\frac{c_0}{c_1}} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\mu_2}{\mu_1} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\mu_2\left(-\frac{1}{v}+\frac{1}{r}\right) = \mu_1\left(-\frac{1}{u}+\frac{1}{r}\right)$

Or $-\frac{\mu_2}{v} + \frac{\mu_2}{r} = -\frac{\mu_1}{u} + \frac{\mu_1}{r}$

Or $\frac{\mu_2 - \mu_1}{r} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$

Or $\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}}$ ……………………………………………….(3.5)

Equation (3.5) gives the equation of refraction for a convex surface. The expression is same as that for a concave surface.

**b) Source or object at the denser medium.**

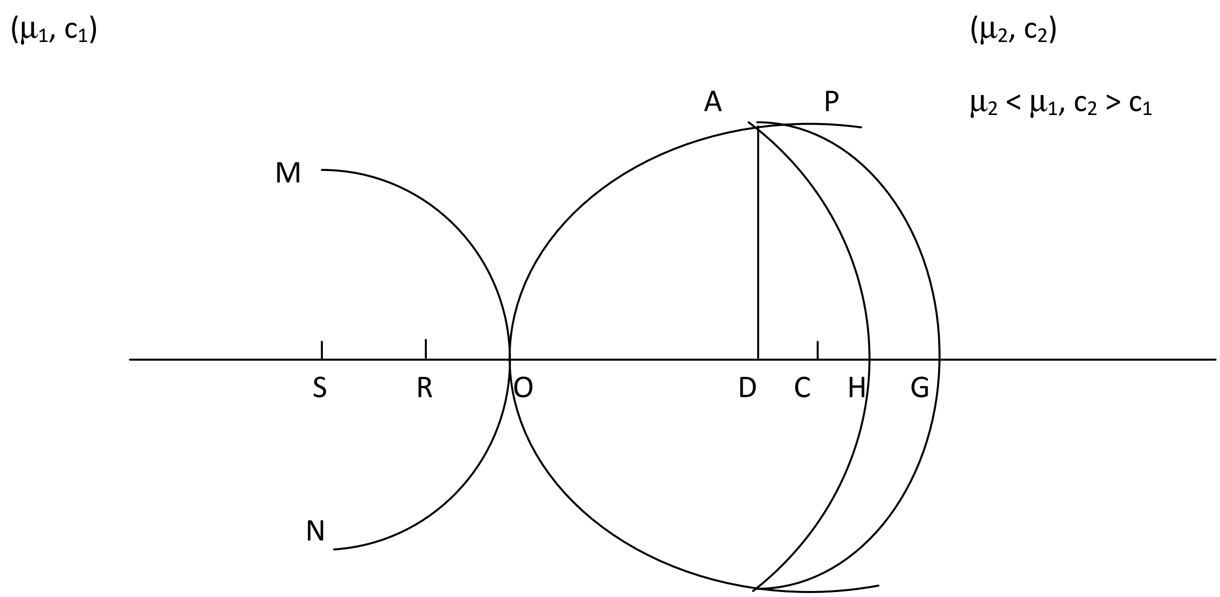



Fig. 3.4: Demonstration of refraction at a convex surface from denser to rarer medium.

PQ $\rightarrow$ concave refracting surface separating two media of refractive indices $\mu_1$ and $\mu_2$

C $\rightarrow$ centre of curvature of MN

OC $\rightarrow$ radius of curvature = r

$c_1 \rightarrow$ velocity of light in medium $\mu_1$

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$R \rightarrow$ centre of the spherical wave front AGB. R is the real image of S

OR $\rightarrow$ Image distance = -v

OG $\rightarrow$ distance travelled by the wave front in time ‘t’ in $\mu_2 = c_2t$

AD $\rightarrow$ perpendicular dropped from A on the principal axis

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$\therefore$ $2OS.HD = AD^2$ ……………………………………(3.2)

In the same way considering arc AGB with centre with same approximations

$2OR.GD = AD^2$ ………………………………………….(3.3)

Now $\frac{c_1 t}{c_2 t} = \frac{HO}{GO} = \frac{HD+OD}{GD+OD} = \frac{\frac{AD^2}{2}\left(\frac{1}{OS}+\frac{1}{OC}\right)}{\frac{AD^2}{2}\left(\frac{1}{OR}+\frac{1}{OC}\right)} = \frac{\frac{1}{OS}+\frac{1}{OC}}{\frac{1}{OR}+\frac{1}{OC}} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$ ……. From (3.1) (3.2) (3.3)

Or $\frac{c_1}{c_2} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\frac{c_0}{c_2}}{\frac{c_0}{c_1}} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\frac{\mu_2}{\mu_1} = \frac{-\frac{1}{u}+\frac{1}{r}}{-\frac{1}{v}+\frac{1}{r}}$

Or $\mu_2\left(-\frac{1}{v}+\frac{1}{r}\right) = \mu_1\left(-\frac{1}{u}+\frac{1}{r}\right)$

Or $-\frac{\mu_2}{v} + \frac{\mu_2}{r} = -\frac{\mu_1}{u} + \frac{\mu_1}{r}$

Or $\frac{\mu_2 - \mu_1}{r} = \frac{\mu_2}{v} - \frac{\mu_1}{u}$

Or $$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}}$$ …………………………………………….(3.5)

Equation (3.5) gives the equation of refraction for a convex surface. The expression is same as that for a concave surface.

**Thus it is found that whatever be the nature of the spherical surface and whatever be the nature of medium on either side of the surface the equation of refraction is always**

$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}}$$