IV SEM -1

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1.INTRODUCTION

The subject of quantum mechanics provides a mathematical description to understand the phenomena which occur on a microscopic (atomic or subatomic) scale. In fact, the subject grew as a result of culmination of new and path-breaking ideas introduced to account for the observed phenomena which could not be explained on the basis of classical concepts. This module and the one following it, present a review of some of the basic concepts and ideas which lay the groundwork for a systematic development of the subject.

BLACK BODY RADIATION.

Radiation is the process of transmission of energy in the form of electromagnetic waves which travel with the speed of light. The energy is transferred directly from the source to the target without requiring any material medium for propagation and without influencing the intervening medium through which it propagates.

Blackbody is an ideal body or surface that completely absorbs all radiant energy falling upon it with no reflection or transmission and that radiates at all frequencies with a spectral energy distribution dependent on its absolute temperature

2.ELECTROMAGNETIC WAVES AND PHOTONS

2.1 Light Quanta and the Planck-Einstein Relation :

One of the earliest phenomena which electromagnetic theory was unable to explain was the nature of the distribution of energy in the spectrum of radiation from a black body. As you know, a black body, by definition, is one which absorbs all the radiation it receives and does not reflect any ('black'!). Such objects are perfect absorbers as well as emitters. A small opening of a hollow enclosure (as for instance, an oven) can be the best practical realization of a black body. Such a cavity with a small aperture through which radiation from outside may be admitted contains radiations emitted by the walls of the enclosure. The spectrum of the radiation is characterized by a function $E(\nu)$, where $E(\nu)d\nu$ represents energy (per unit volume of the cavity) of the radiation with frequencies between ν and $\nu+d\nu$..

Experimental observations, as depicted in Fig.(1.1), show that the spectral distribution of black body radiation, which depends only on the temperature, plotted as a function of frequency ν falls off after reaching a maximum, as frequency is increased.

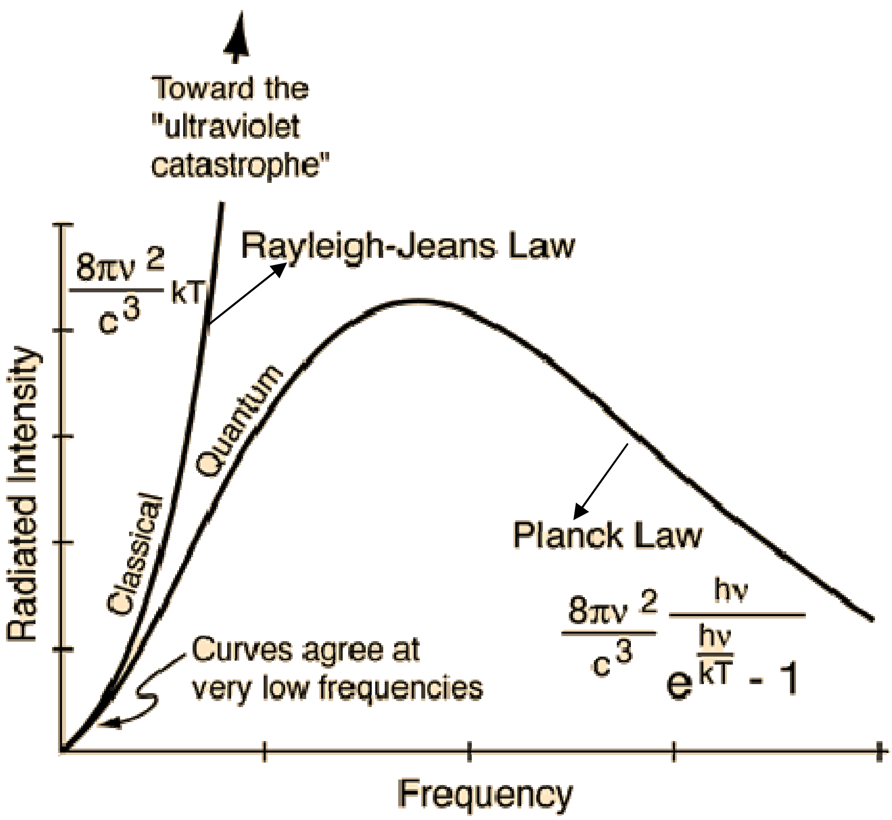

Fig. 1.1

Earlier attempts based on classical ideas were found to be inadequate to explain the observed frequency distribution of the black body. Thus, for instance, Wien suggested semi-empirical theory which agreed only in the short wavelength limit, while Raleigh and Jeans deduced, from classical reasoning, a law which agreed at long wavelengths limit but was in complete disagreement for short wavelengths. The basis of their argument was that according to classical electromagnetic theory, the energy density distribution, E(ν), of a black body radiation can be expressed as

$$E(\nu) = \frac{8\pi \nu^2}{c^3} U(\nu) \quad \ldots\ldots\ldots\ldots\ldots\ldots(1.1)$$

Where $\frac{8\pi \nu^2}{c^3}$ is the number of electromagnetic oscillators per unit volume at frequency ν in the range dν in equilibrium with the radiation in a black body cavity and u(ν) represents the average energy of the oscillators. From the 'Law of Equipartition of Energy', the mean energy of the oscillators with frequency ν is

$$u(\nu)=k\,T\,,$$

where k is the Boltzmann's constant, and T is the absolute temperature. Thus we get

$$E(\nu)d\nu = \frac{8\pi\nu^2}{c^3}\,KTd\nu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.2)$$

Or

$$E(\lambda)d\lambda = \frac{8\pi}{\lambda^4}\,KTd\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.3)$$

This is known as Rayleigh-Jean's law. According to this law the energy radiated in a given wavelength range dλ increases indefinitely as λ becomes smaller and smaller. This is clearly in complete disagreement with the observed spectral distribution.

To explain the observed features, it led Planck to suggest the **hypothesis of the quantization of energy (1900)**:- For an electromagnetic wave of frequency ν, the only possible energies are integral multiples of the quantum of energy, **ε= hν**, where 'h' is called the Plancks constant. Planck did not introduce the concept of photons. He, however, suggested the emission and absorption of radiation by matter takes place in discrete quanta of energy.

Planck also assumed that a black body is composed of oscillators in equilibrium with radiation field. The number of oscillators with energy in equilibrium with temperature T can be obtained from the classical Boltzmann expression and is given by:

$$N(\nu) = e^{-\frac{\epsilon_n}{KT}} \sum_{n=0}^{\infty} e^{-\frac{\epsilon_n}{KT}} \quad \ldots\ldots(1.4)$$

where N(ν) is the number of oscillators of frequency ν . The mean energy per oscillator of frequency is

$$U(\nu) = \frac{\sum_{n=0}^{\infty} \epsilon_n e^{-\frac{\epsilon_n}{KT}}}{\sum_{n=0}^{\infty} e^{-\frac{\epsilon_n}{KT}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.5)$$

Where ϵ_n = hν. Now $\sum_{n=0}^{\infty} e^{-\frac{h\nu}{KT}} = 1 + e^{-\frac{h\nu}{KT}} + e^{-\frac{2h\nu}{KT}} + \ldots\ldots$

$$= \left(1 - e^{-\frac{h\nu}{KT}}\right)^{-1} \quad \ldots\ldots\ldots(1.6)$$ (summing a GP where common ratio is < 1)

Similarly it can be shown that

$$\sum_{n=0}^{\infty} \epsilon_n\, e^{-\frac{\epsilon_n}{KT}} = \frac{h\nu e^{-\frac{h\nu}{KT}}}{\left(1- e^{-\frac{h\nu}{KT}}\right)^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.7)$$

From equations (1.5), (1.6), (1.7) we have

$$U(\nu) = \frac{h\nu e^{-\frac{h\nu}{KT}}}{\left(1 - e^{-\frac{h\nu}{KT}}\right)} = \frac{h\nu}{\left(e^{\frac{h\nu}{KT}}\right)} \quad \ldots\ldots\ldots\ldots\ldots(1.8)$$

We thus get Planck's expression for the density of radiation from equation (1.1) and (1.8) as

$$E(\nu) = \frac{8\pi\nu^2}{c^3} U(\nu) = \frac{8\pi\nu^3}{c^3} \frac{h}{\left(e^{\frac{h\nu}{KT}} - 1\right)} \quad \ldots\ldots\ldots(1.9)$$

which agrees very closely with the observed radiation distribution.

In the long wavelength (or low frequency) limit, where hν << KT we can write

$$e^{\frac{h\nu}{KT}} - 1 = 1 + \frac{h\nu}{KT} - 1 = \frac{h\nu}{KT}$$

This gives: $E(\nu) = \frac{8\pi\nu^2}{c^3} KT$

Which is, in accordance with **Raleigh Jeans Law.**

In the short wavelength or high frequency region where hν >> KT we can write

$e^{\frac{h\nu}{KT}} >> 1$. Hence $e^{\frac{h\nu}{KT}} - 1 \approx e^{\frac{h\nu}{KT}}$. This **gives** $E(\nu = \frac{8\pi h\nu^3 e^{-\frac{h\nu}{KT}}}{c^3}$ …………(1.10)

Which is in accordance with Wien's law.

Total Radiation density

The total radiation density can be obtained from Eq.(1.9) by integrating over all frequencies,

$$E = \int_{\nu=0}^{\infty} E(\nu) d\nu = \int_{\nu=0}^{\infty} \frac{8\pi\nu^3}{c^3} \frac{h}{\left(e^{\frac{h\nu}{KT}} - 1\right)} = \frac{8\pi^5 K^4 T^4}{15c^3 h^3} \quad \ldots\ldots\ldots\ldots\ldots(1.11)$$

showing that the energy density is proportional to the 4th power of the absolute temperature. This is known as **Stefan's Law** – suggested first by Stefan.