TEMPORAL COHERENCE OR COHERENCE IN TIME

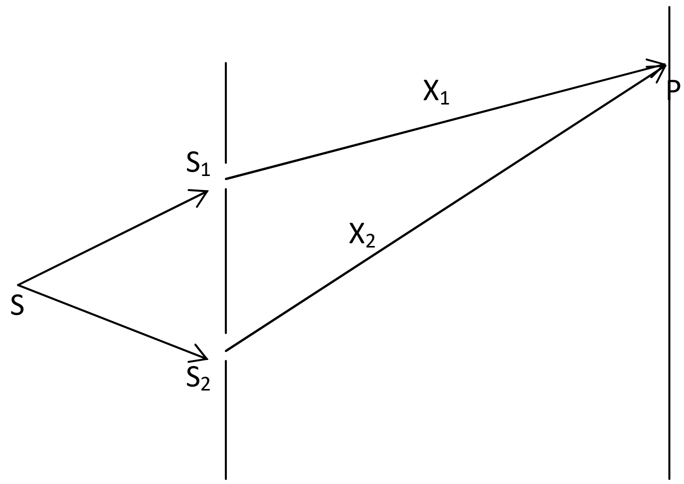

Fig. 1.1. Schematic diagram of Young'S double slit experiment.

S_1 and S_2 are two slits equidistant from a very narrow source S. S_1 and S_2 divides the wave front coming from S and the two divided parts being coherent interfere. P is a point of interference. $S_1P = x_1$ and $S_2P = x_2$. So **path difference between the interfering waves is $\Delta x = x_2 \sim x_1$**. Let it be assumed that $x>x_1$. Now a light pulse is emitted from the source when source atoms undergo transition from higher to lower energy levels. The excess radiation is emitted in the form of wave. Let τ_c be the time between successive transitions i.e. after each interval of τ_c a wave pulse is emitted from S. **τ_c is called the coherence time. The distance travelled by the wave in time τ_c is called the coherence length and is equal to $L_c = c\tau_c$**, where c is the velocity of light. If **$\Delta x >> L_c$,** then the wave from S_1 will reach P considerably before the wave from S_2. If 't' be the additional time taken by the wave from S_2 to reach P then **: $t = \frac{\Delta x}{c}$. As $\Delta x >> L_c$, hence $t >> \tau_c$**. hence within the time 't' several other transition will occur in S and consequently other wave pulses are generated. It may so happen that at the instance the wave originating from the first transition reaches P from S_2, the wave originating from some other transition reaches there from S_1 So interference occurs between two waves emitted from the source at two different instants. So they are not coherent. Under such circumstance interference is said to have lost **Temporal coherence. So the condition for temporal coherence is : $t \leq \tau_c$.** Temporal coherence is a manifestation of spectral purity. τ_c gives a measure of temporal coherence. Higher the value of τ_c, better will be the temporal coherence.

SPATIAL COHERENCE OR COHERENCE IN SPACE.

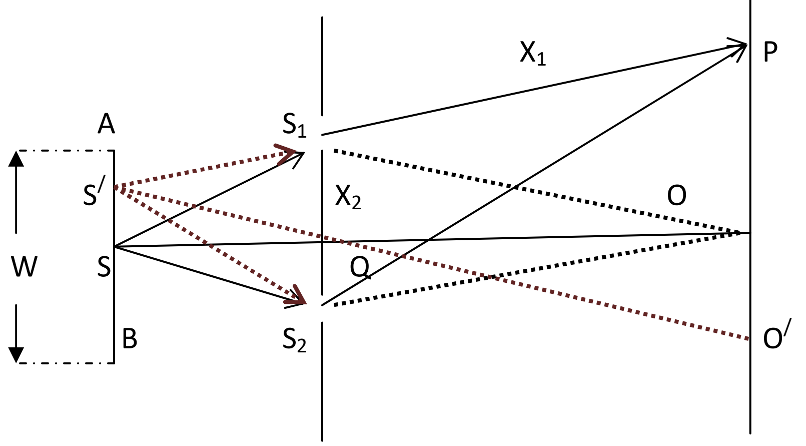

Fig 1.2: Schematic diagram to explain spatial coherence.

Let it be assumed that instead of being a narrow source, the source slit AB is an extended one of width 'W . So each section of the wide slit can be considered at a separate narrow slit i.e the wide slit can be considered as a collection of a number of independent narrow slits. Let S and S' be two such narrow slits. O is a point such that $SS_1 + S_1O = SS_2 + S_2O$. So path difference between the interfering beams coming from S is zero at O. So S forms its own fringe pattern with zero order maxima at O. O' is a point such that $S'S_1 + S_1O = S'S_2 + S_2O$. So path difference between the interfering beams coming from S' is zero at O. So S' also forms its own fringe pattern with zero order maxima at O'. Similarly each of the component slits will form their own individual interference pattern with zero orders at different positions on the screen. There is no guaranty that all of them will form maximas and minimas at same positions of the screen. It may so happen that maximas of some pattern overlap on the minima of another. Due to such multiple overlapping, the screen will exhibit uniform illumination instead of a distinct pattern. **Under such circumstance it is said that there is no Spatial coherence .** For interference to have spatial coherence, the source slit should be as narrow as possible.

Mathematical analysis of spatial coherence.

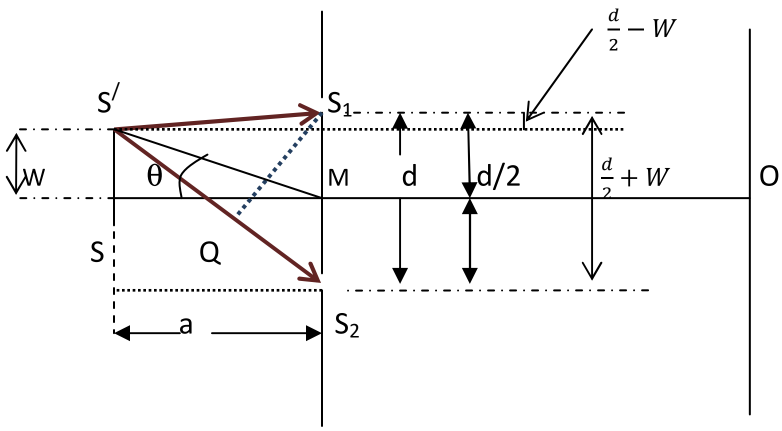

Fig. 3.3 -

Let SS/ be an extended light source of width 'W'. The two terminal points of the source – S and S/ can be considered as two independent slits which illuminates the double slit arrangement. They form their own individual interference pattern which overlaps on one another on the screen. It is required to find the least value of 'W' for which the pattern on the screen disappears. This will occur when the maximas due to S coincides with the minima due to S/ and vice versa. Assuming S to be equidistant from S_1 and S_2 , O is the position of the zero order bright fringe on the screen due to S.

a = separation between the source and the double slit.

d = separation between S_1 and S_2.

Q is a point such that $S'S_1 = S'Q$. So the wave from S/ reaching O, via S_1 and S_2 have a path difference QS_2. O being equidistant from S_1 and S_2, a dark fringe will be formed there due to S/ if the minimum value of **$QS_2 = \frac{\lambda}{2}$** , λ being the wavelength of the light from the source. When this occurs the maximas due to waves from S are destroyed by the minimas due to waves from S/.

From the fig. (3.3) :

$$S'S_1 = \left\{a^2 + \left(\frac{d}{2} - W\right)^2\right\}^{\frac{1}{2}} = a\left\{1 + \frac{1}{a^2}\left(\frac{d}{2} - W\right)^2\right\}^{\frac{1}{2}}$$

expanding binomially and neglecting higher order terms under the assumption : a >>d and W.

$$S'S_1 = a\left\{1 + \frac{1}{a^2}\left(\frac{d}{2} - W\right)^2\right\}$$

$$= a + \frac{1}{2a}\left(\frac{d}{2} - W\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.1)$$

Similarly:: $S^{/}S_2 = a + \frac{1}{2a}\left(\frac{d}{2} + W\right)^2$ ……………………(3.2)

From equations (3.1) and (3.2): $S^{/}S_2 - S^{/}S_1 = QS_2 = \frac{Wd}{a}$ ……………………(3.3)

For the fringes to disappear the minimum value should be $\frac{\lambda}{2}$

So $\left(\frac{Wd}{a}\right)_{min} = \frac{\lambda}{2}$

As 'd' and 'a' are constants in a particular arrangement:

$$(W)_{min} = \frac{\lambda a}{2d}. \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.4)$$

It follows that if the double slit is illuminated by a source of width exceeding $\frac{\lambda a}{2d}$, no interference pattern will be obtained on the screen. If we have an extended source of width $\frac{\lambda a}{d}$, then for every point on the source there will be a corresponding point at a distance of $\frac{\lambda a}{2d}$ which destroys each other's pattern and no fringes will be obtained on the screen So for an extended source fringes of good contrast will be observed only if :

$$\boxed{W \ll \frac{\lambda a}{d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.5)$$

Equation (3.5) gives the condition for good spatial coherence.

If the angle subtended by the source $SS^{/}$ at M(Midpoint of S_1S_2 be 'θ' then : $\theta = \frac{W}{a}$ and equation (3.5) becomes**:** **$d \ll \frac{\lambda}{\theta}$, which is an alternative condition of good spatial coherence.**

The distance $\frac{\lambda}{\theta}$ denoted by **'l_w'** gives the lateral distance over which the light beam may be assumed to be spatially coherent and it is known as **lateral coherence width**. For a circular source 'l_w' is given by : $l_w = 1.22\frac{\lambda}{\theta}$