Diffraction III

Circular aperture

Double slit

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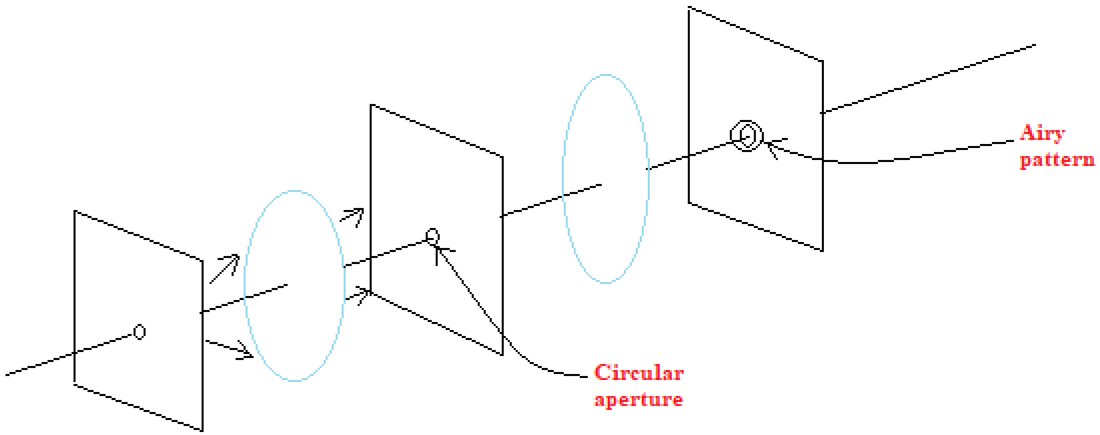
CIRCULAR APERTURE

A plane wave is incident normally on the circular aperture
Fraunhofer diffraction pattern is observed in the focal plane of the lens
Due to rotational symmetry of the system, the diffraction pattern will consists of concentric bright and dark rings.
This pattern is known as the Airy pattern

The angular spread of the beam is approximately

$$\Delta\theta \cong 1.22\,\frac{\lambda}{d}$$

Where d is the diameter of the aperture
If the two stars has angular separation $\Delta\phi$ and
$\Delta\phi \gg \Delta\theta$, the images of the two stars will be formed on the focal plane of the telescope.



According to Rayleigh's criterion, the stars are said to be just resolved when the centre of one Airy's disk falls on the first minimum of the Airy pattern of the other star

Limit of resolution is $(\Delta\phi)_{min} = \Delta\theta = 1.22\frac{\lambda}{d}$

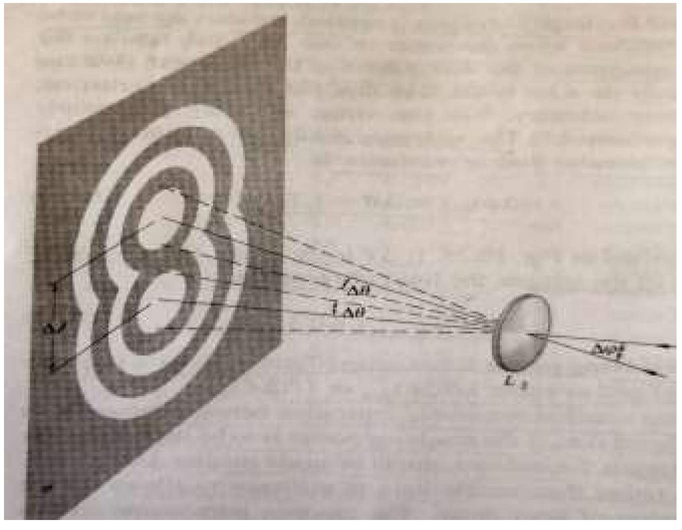
If Δl is the centre-to-centre separation of the image, the limit of resolution is

$$(\Delta l)_{min} = 1.22\frac{f\lambda}{d}$$

The resolving power of any image forming system is generally defined as

$$\frac{1}{(\Delta\phi)_{min}} \text{ or } \frac{1}{(\Delta l)_{min}}$$

Considering human eyes



$d = 2mm$, and

$\lambda = 550nm$

$$\Delta\theta = \frac{1.22 \times 550\ nm}{2\ mm}$$

$$= \frac{1.22 \times 550 \times 10^{-9}}{2 \times 10^{-3}} \text{ radian}$$

$$= 0.61 \times 550 \times 10^{-6} \times \frac{180}{\pi} \times 60 \text{ arc-min}$$

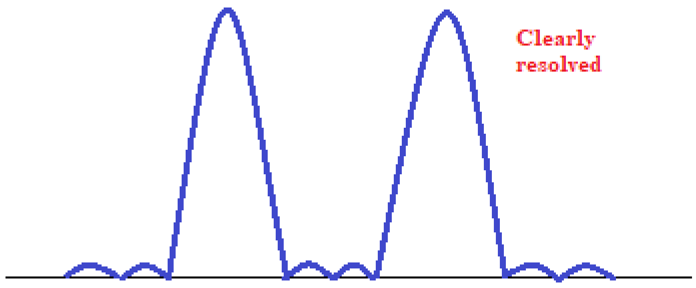
$$\sim 1\ arc - min$$

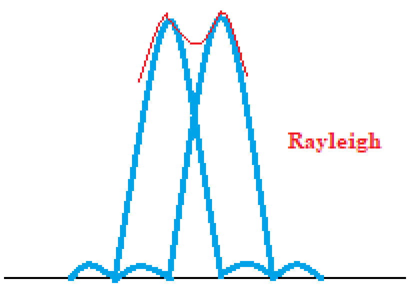
If $f = 20\, mm$, then

$$(\Delta l)_{min} = 0.61 \times 550 \times 10^{-6} \times 20$$
$$\sim 6700 nm$$

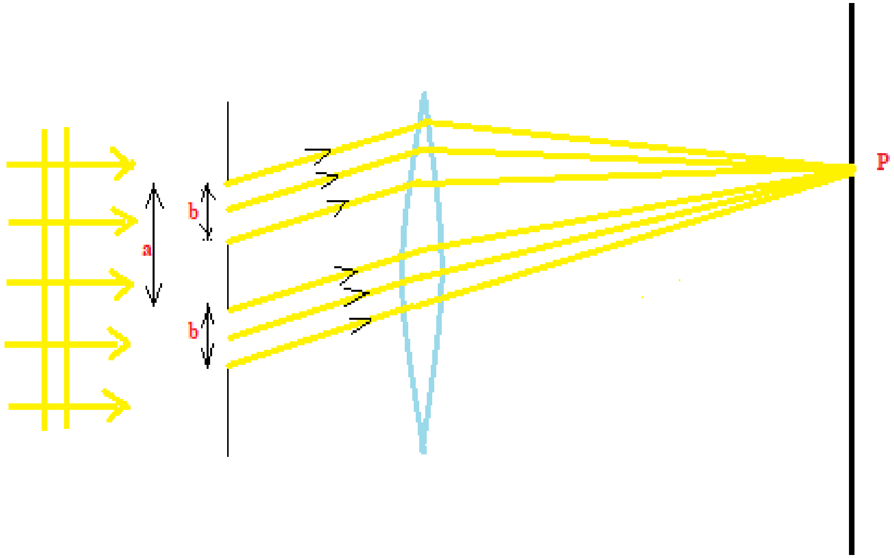
Consider Hubble space telescope

$$d \sim 2.4 m$$
$$f \sim 57.6 m$$
$$\Delta\phi = \frac{1.22 \times 550 nm}{2.4\, m}$$
$$= \frac{1.22 \times 550 \times 10^{-9}}{2.4} \times \frac{180}{\pi} \times 60 \times 60\ arc-sec$$
$$\sim 0.5\ arc-sec$$





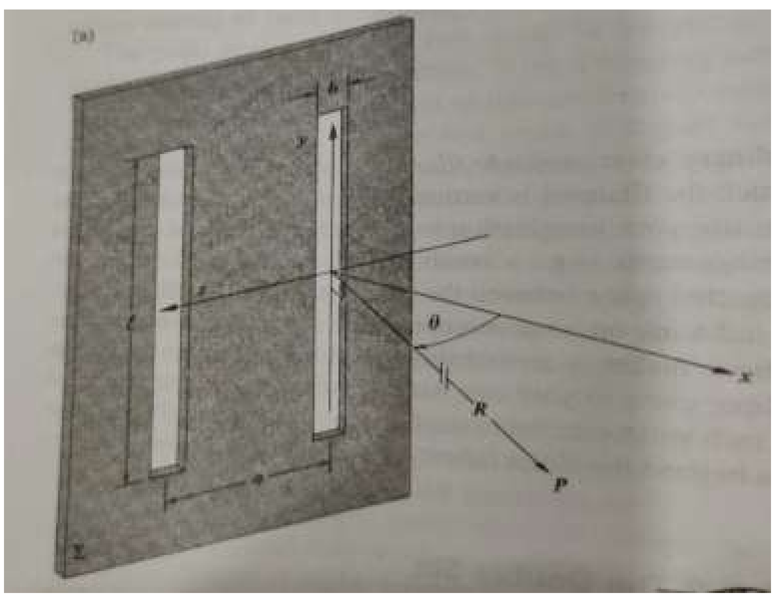
DOUBLE SLIT



If the diffracted rays make an angle θ with the normal to the plane of the slits, then the path difference between the disturbances at a point P will be $\Delta sin\theta$, and corresponding phase difference

$$\phi = \frac{2\pi}{\lambda}\Delta sin\theta$$

Let each of the two aperture or slit is divided into differential strips dz along z axis



The total contribution to the electric field, in the Fraunhofer approximation,

$$\tilde{E} = \widetilde{E_0}\int_{-b/2}^{b/2} e^{ikz}dz + \widetilde{E_0}\int_{a-b/2}^{a+b/2} e^{ikz}dz$$

$$= A\frac{sin\beta}{\beta}cos\alpha\cos(\omega t - \frac{\beta}{2} - \frac{\phi}{2})$$

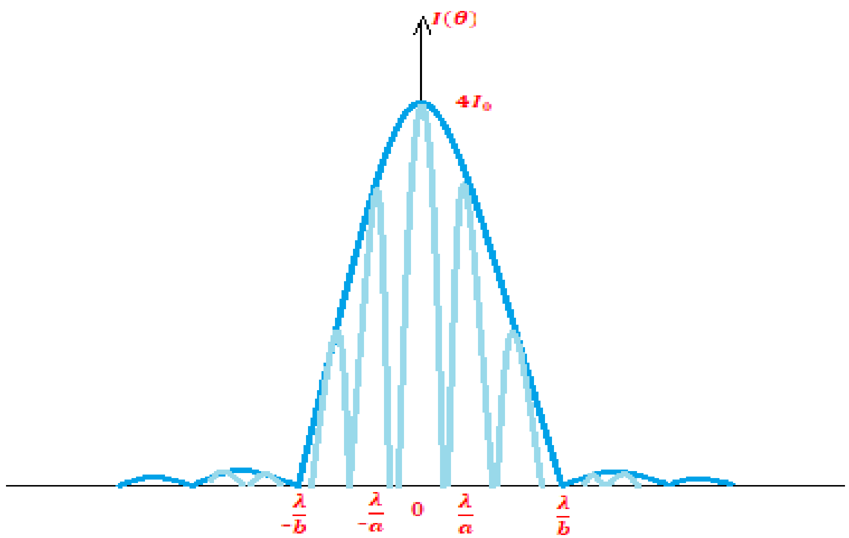
$\alpha = \frac{\pi}{\lambda} a \sin\theta$

and $\beta = \frac{\pi}{\lambda} b \sin\theta$

$2\alpha = \frac{2\pi}{\lambda} a \sin\theta$ = phase difference between two waves arriving at P, one having originated at any point in the first slit, and the other coming from the corresponding point in the second slit.

$2\beta = \frac{2\pi}{\lambda} b \sin\theta$ = phase difference between two nearly parallel rays, arriving at a point P on the screen from edges of one of the slits

When squared and averaged over a relatively long interval in time the intensity



$$I(\theta) = 4I(0)\left(\frac{\sin^2\beta}{\beta^2}\right)\cos^2\alpha$$

$$= 4I(0)\mathrm{sinc}^2\beta \cos^2\alpha$$

If $\theta = 0, \alpha = \beta = 0$

$I(0) = 4I_0$, I_0 is the intensity distribution from either slit

If $kb \ll 1$ i.e. β is very small and we assume $\frac{sin\beta}{\beta} \approx 1$ then

$$I(\theta) = 4I(0)cos^2\alpha$$

This gives the expression for intensity distribution of Young's Experiment

If $\alpha = 0$, and $\alpha = 0$, then $I(0) = 4I_0 \frac{sin^2\beta}{\beta^2}$

This gives the expression for single slit diffraction with doubled the source strength

Then we might envision the total expression as being generated by a $cos^2\alpha$ interference term modulated by $sinc^2\beta$ diffraction term

We have $\beta = \frac{\pi}{\lambda}\, b\, sin\theta$, and $\alpha = \frac{\pi}{\lambda}\, a\, sin\theta$

If we put $\beta = m\pi$, then the condition for diffraction minima will be achieved

$$b\, sin\theta = m\lambda$$

$$a \sin\theta = (n + \frac{1}{2})\lambda$$

The interference maxima occur when

$$\alpha = 0, \pi, 2\pi, \ldots$$

Or when $a \sin\theta = 0,\ \lambda, 2\lambda, \ldots$

The actual position of the maxima will approximately occur at the above angles provided the variation of the diffraction term is not too rapid

Also a maximum may not occur at all if θ corresponds to a diffraction minimum

Let for a slit

$$b = 0.0088\ cm$$
$$\lambda = 6000\dot{A}$$

The 1st diffraction minimum will occur at

$$\theta = \frac{\lambda}{b} = \frac{6\times 10^{-5}}{8.8\times 10^{-3}}\ radian$$
$$= 0.0068\ radian$$
$$= 0.389^0$$

This is the first minimum of the diffraction term, so interference maxima are extremely weak around $\theta \sim 0.4^0$

Let us take $b = 8.8 \times 10^{-3}\ cm$

$a = 7.0 \times 10^{-2}\ cm$, and

$\lambda = 6000\dot{A}$

How many interference minima will occur between two diffraction minima on either side of the central maximum?