APPLICATION OF HUYGEN'S CONSTRUCTION

I. TO SHOW THAT FOR REFLECTRION IN A PLANE MIRROR THE OBJECT DISTANCE IS EQUAL TO THE IMAGE DISTANCE.

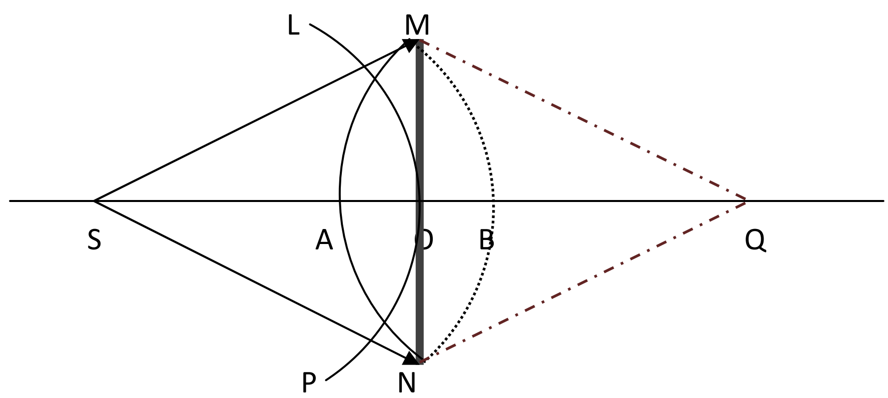

Fig. 3.1. Reflection in a plane mirror

Let light travel from a source through a homogenous and isotropic medium.

MN $\rightarrow$ Plane mirror

S $\rightarrow$ Source or object

O $\rightarrow$ Midpoint of MN

OS $\rightarrow$ Object distance

LOP $\rightarrow$ Incident spherical wave front with centre S

MBN $\rightarrow$ Position of spherical wave front centered at S after time, 't' if the mirror hadn't been there.

OB = ct, where 'c' is the velocity of light in the concerned medium

MAN $\rightarrow$ Position of the spherical wave front centered at Qforthe wave reflected from the mirror after time, 't'

OQ $\rightarrow$ Image distance

OA = ct , as the wave returns to the same medium through which it was incident.

Since the medium is homogenous the reflected wave front is also spherical with centre at Q. So Q is the image of S.

From the geometry of the figure: Considering the arc MBN

$$(2SB - OB).OB = MO^2$$

Assuming the distance SO to be large compared to the dimension of the mirror : OA << OS, which gives-

$$2OS.OB = MO^2 \quad \text{...(3.1)}$$

Similarly considering arc MAN

$$2OQ.OA = MO^2 \quad \text{..(3.2)}$$

Now OA = OB = ct ....(3.3)

From equations (3.1), (3.2) and (3.3)

OS = OQ

Or **object distance = image distance**

II . TO FIND THE EQUATION OF REFLECTION AT A SPHERICAL SURFACE.

a) Reflection at concave spherical surface

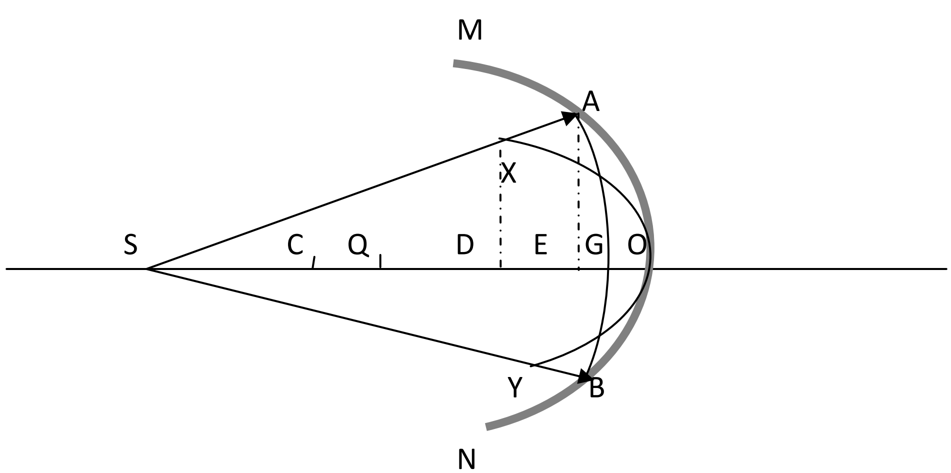

Fig. 3.2. Reflection at a concave spherical surface

Let light travel from a source through a homogenous and isotropic medium.

MN $\rightarrow$ Concave mirror centered at C

OC $\rightarrow$ Radius of curvature = -r

S $\rightarrow$ Source or object

O $\rightarrow$ Midpoint of MN

OS $\rightarrow$ Object distance = -u

XOY$\rightarrow$ Incident spherical wave front with centre S

AGB $\rightarrow$ Position of the spherical wave front centered at Q for the wave reflected from the mirror after time, 't'

OQ $\rightarrow$ Image distance = -v

XD, AE $\rightarrow$ Perpendiculars, from X and A respectively, on the principal axis. For a mirror of small aperture and large radius of curvature **XD ≈ AE** **………………………………**(3.4)

OG = XA = YB = DE = ct, where 'c' is the velocity of light in the concerned medium

From the geometry of the figure: Considering the arc XOY

$$(2OS - OD).OD = XD^2$$

For a mirror of large radius of curvature and small aperture : OD << OS, which gives:

$$2OS.OD = XD^2$$

Or $$OD = \frac{XD^2}{2OS} = -\frac{XD^2}{2u} \quad \text{……………………………………}(3.5)$$

Similarly, considering AGB

$$2GS.GE = AE^2$$

As AGB has a large radius so compared to it the points G and O are almost overlapping. So GS = OS and, which gives:

$$2OQ.GE = AE^2$$

Or $$GE = \frac{AE^2}{2OQ} = -\frac{AE^2}{2v} \quad \text{……………………………………………}(3.6)$$

Similarly considering arc AOB

2OC.OE = AE^2

Or $\quad OE = \frac{AE^2}{2OC} = -\frac{AE^2}{2r}$ ……………………………………………….(3.7)

Now: OD = OE + ED = OE + OG = OE + (OE – GE)

Or $\quad OD + GE = 2OE$

So from equations (3.5), (3.6) and (3.7)

Or $\quad -\frac{XD^2}{2u} - \frac{AE^2}{2v} = -\frac{2AE^2}{r}$

Applying equation (3.4)

$$-\frac{AE^2}{2u} - \frac{AE^2}{2v} = -\frac{2AE^2}{2r}$$

Or $\quad -\frac{1}{2u} - \frac{1}{2v} = -\frac{2}{2r}$

O $\quad -\frac{1}{u} - \frac{1}{v} = -\frac{2}{r}$

Or $\quad \boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{r}}$ ……………………………………………….(3.8)

Equation (3.8) gives the equation of reflection for a concave reflecting surface.

a) Reflection at convex spherical surface

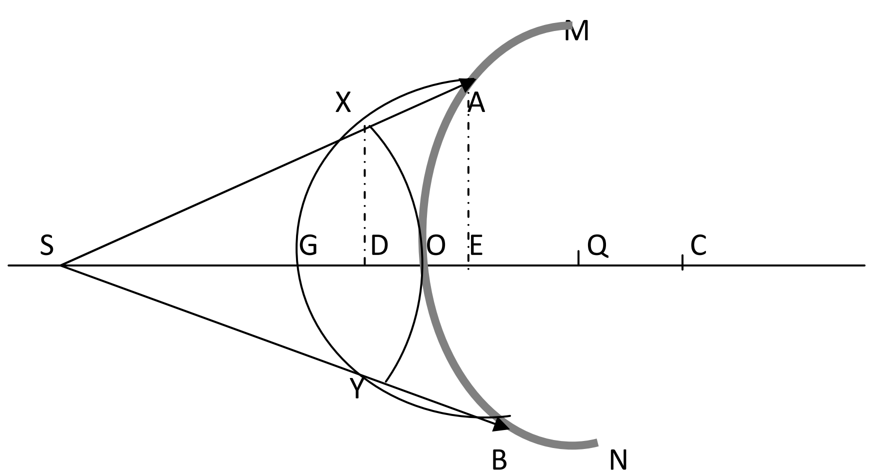

Fig. 3.2. Reflection at a concave spherical surface

MN $\rightarrow$ Convex mirror centered at C

OC $\rightarrow$ Radius of curvature = r

S $\rightarrow$ Source or object

O $\rightarrow$ Midpoint of MN

OS $\rightarrow$ Object distance = -u

XOY$\rightarrow$ Incident spherical wave front with centre S

AGB $\rightarrow$ Position of the spherical wave front centered at Q for the wave reflected from the mirror after time, ‘t’

OQ $\rightarrow$ Image distance = v

XD, AE $\rightarrow$ Perpendiculars, from X and A respectively, on the principal axis. For a mirror of small aperture and large radius of curvature **XD ≈ AE** **……………………………….**(3.9)

OG = XA = YB = DE = ct, where ‘c’ is the velocity of light in the concerned medium.

From the geometry of the figure: Considering the arc XOY

$$(2OS - OD).OD = XD^2$$

For a mirror of large radius of curvature and small aperture : OD << OS, which gives:

$$2OS.OD = XD^2$$

Or $$OD = \frac{XD^2}{2OS} = -\frac{XD^2}{2u} \quad \text{……………………………………..(3.10)}$$

Similarly, considering AGB

$$2GS.GE = AE^2$$

As AGB has a large radius so compared to it the points G and O are almost overlapping. So GS = OS and, which gives:

$$2OQ.GE = AE^2$$

Or $$GE = \frac{AE^2}{2OQ} = \frac{AE^2}{2v} \quad \text{…………………………………………….(3.11)}$$

Similarly considering arc AOB

$$2OC.OE = AE^2$$

Or $\quad OE = \frac{AE^2}{2OC} = \frac{AE^2}{2r}$ ……………………………………………….(3.12)

Now: $\quad$ OD = ED – OE = OG – OE = (GE – OE) – OE

Or $\quad$ OD – GE = -2OE

So from equations (3.10), (3.11) and (3.12)

$$-\frac{XD^2}{2u} - \frac{AE^2}{2v} = -\frac{2AE^2}{2r}$$

Applying equation (3.9)

$$-\frac{AE^2}{2u} - \frac{AE^2}{2v} = -\frac{2AE^2}{2r}$$

Or $\quad -\frac{1}{2u} - \frac{1}{2v} = -\frac{2}{2r}$

O $\quad -\frac{1}{u} - \frac{1}{v} = -\frac{2}{r}$

Or $\quad \boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{r}}$ ……………………………………………….(3.13)

Equation (3.8) gives the equation of reflection for a convex reflecting surface.

Thus it is found that whatever be the nature of the spherical reflecting surface the equation of reflaction is always :

$$\boxed{\frac{1}{u} + \frac{1}{v} = \frac{2}{r}}$$