MAGNETISM IN SOLIDS

History:

Magnetism was first discovered in the ancient world, when people noticed that lodestones, naturally magnetized pieces of the mineral magnetite, could attract iron. The word *magnet* comes from the Greek term meaning "the Magnesian stone, lodestone." In ancient Greece, Aristotle attributed the first of what could be called a scientific discussion of magnetism to the philosopher Thales of Miletus, who lived from about 625 BC to about 545 BC. The ancient Indian medical text *Sushruta Samhita* describes using magnetite to remove arrows embedded in a person's body.

In ancient China, the earliest literary reference to magnetism lies in a 4th-century BC book named after its author, *The Sage of Ghost Valley*. The 2nd-century BC annals, *Lüshi Chunqiu*, also notes: "The lodestone makes iron approach, or it attracts it." The earliest mention of the attraction of a needle is in a 1st-century work *Lunheng* (*Balanced Inquiries*): "A lodestone attracts a needle." The 11th-century Chinese scientist Shen Kuo was the first person to write—in the *Dream Pool Essays*—of the magnetic needle compass and that it improved the accuracy of navigation by employing the astronomical concept of true north. By the 12th century, the Chinese were known to use the lodestone compass for navigation. They sculpted a directional spoon from lodestone in such a way that the handle of the spoon always pointed south.

Alexander Neckam, by 1187, was the first in Europe to describe the compass and its use for navigation. In 1269, Peter Peregrinus de Maricourt wrote the *Epistola de magnete*, the first extant treatise describing the properties of magnets. In 1282, the properties of magnets and the dry compasses were discussed by Al-Ashraf, a Yemeni physicist, astronomer, and geographer.[9]

Leonardo Garzoni's only extant work, the *Due trattati sopra la natura, e le qualità della calamita*, is the first known example of a modern treatment of magnetic phenomena. Written in years near 1580 and never published, the treatise had a wide diffusion. In particular, Garzoni is referred to as an expert in magnetism by Niccolò Cabeo, whose Philosophia Magnetica (1629) is just a re-adjustment of Garzoni's work. Garzoni's treatise was known also to Giovanni Battista Della Porta and William Gilbert.

An understanding of the relationship between electricity and magnetism began in 1819 with work by Hans Christian Oersted, a professor at the University of Copenhagen, who discovered by the accidental twitching of a compass needle near a wire that an electric current could create a magnetic field. This landmark experiment is known as Oersted's Experiment. Several other experiments followed, with André-Marie Ampère, who in 1820 discovered that the magnetic field circulating in a closed-path was related to the current flowing through a surface enclosed by the path; Carl Friedrich Gauss; Jean-Baptiste Biot and Félix Savart, both of whom in 1820 came up with the Biot–Savart law giving an equation for the magnetic field from a current-carrying wire; Michael Faraday, who in 1831 found that a time-varying magnetic flux through a loop of wire induced a voltage, and others finding further links between magnetism and electricity. James Clerk Maxwell synthesized and expanded these insights into Maxwell's

equations, unifying electricity, magnetism, and optics into the field of electromagnetism. In 1905, Albert Einstein used these laws in motivating his theory of special relativity,[10] requiring that the laws held true in all inertial reference frames.

Electromagnetism has continued to develop into the 21st century, being incorporated into the more fundamental theories of gauge theory, quantum electrodynamics, electroweak theory, and finally the standard model.

Sources:

Magnetism, at its root, arises from two sources:

1. Electric current.
2. Spin magnetic moments of elementary particles.

The magnetic properties of materials are mainly due to the magnetic moments of their atoms' orbiting electrons. The magnetic moments of the nuclei of atoms are typically thousands of times smaller than the electrons' magnetic moments, so they are negligible in the context of the magnetization of materials. Nuclear magnetic moments are nevertheless very important in other contexts, particularly in nuclear magnetic resonance (NMR) and magnetic resonance imaging (MRI).

Ordinarily, the enormous number of electrons in a material are arranged such that their magnetic moments (both orbital and intrinsic) cancel out. This is due, to some extent, to electrons combining into pairs with opposite intrinsic magnetic moments as a result of the Pauli exclusion principle (see *electron configuration*), and combining into filled subshells with zero net orbital motion. In both cases, the electrons preferentially adopt arrangements in which the magnetic moment of each electron is canceled by the opposite moment of another electron. Moreover, even when the electron configuration *is* such that there are unpaired electrons and/or non-filled subshells, it is often the case that the various electrons in the solid will contribute magnetic moments that point in different, random directions so that the material will not be magnetic.

Sometimes, either spontaneously, or owing to an applied external magnetic field—each of the electron magnetic moments will be, on average, lined up. A suitable material can then produce a strong net magnetic field.

The magnetic behavior of a material depends on its structure, particularly its electron configuration, for the reasons mentioned above, and also on the temperature. At high temperatures, random thermal motion makes it more difficult for the electrons to maintain alignment.

Magnetic Terminology:

1) Magnetic field strength or intensity(H): The ability of a **magnetic** field to magnetize a material medium is called its **magnetic intensity 'H'**. Its magnitude is measured by the number of ampere-turns flowing round unit length of a solenoid, required to produce that **magnetic** field. Unit of '**H'** is ampere-turns per meter (Am^{-1}).**It is a vector quantity pertaining to the condition at any point under magnetic influence (as of a magnet, an**

electric current, or an electromagnetic wave) measured by the force exerted in vacuum upon a free unit north pole placed at the point in question.

2) Magnetisation(M): When a solid is placed in a magnetic field it gets magnatised. **The magnetic moment per unit volume developed in the solid is called Magnetisation and denoted by a vector 'M'.** Unit of '**M**' is also ampere-turns per meter (Am^{-1}).

3) Magnetic susceptibility(χ): Magnetic susceptibility is a measure of the quality of the magnetic material i.e. the ease with which it can be magnetized. **It is defined as the magnetization produced in the solid per unit applied magnetic field**

$$\chi = \frac{M}{H} \quad \text{...(1.1)}$$

As the vectors '**M**' and '**H**' may have different directions, hence 'χ' is a tensor. However is isotropic media '**M**' and '**H**' point in the same direction and 'χ' is a scalar quantity. Since both '**M**' and '**H**' have same units hence 'χ' is a dimensionless quantity. If the amount of system is one gram molecule of any substance then the susceptibility is termed as **molecular susceptibility (χ_m).** The magnitude and sign of susceptibility vary with the nature of magnetism.

4) Magnetic induction or magnetic flux density(B): Magnetic induction '**B**' is produced inside a medium as a consequence of applied magnetic field '**H**'. '**B**' **is related to** '**H**' **through the relation :**

$$B = \mu_0(H + M) \quad \text{..(1.2)}$$

Where 'μ_0' is the permeability of free space or vacuum and is equal to **$4\pi \times 10^{-7}$ Henry per metre.** The quantity '**B**' is measured in Weber per square metre(Wbm^{-1}) or **Tesla(T).** From equation (1.1) and (1.2) :

$$B = \mu_0(1 + \chi)H \quad \text{...(1.3)}$$

Or $\quad B = \mu H \quad \text{..(1.4)}$

Where 'μ' is called the absolute permeability of the medium. Like 'χ', 'μ' is also, in general a tensor. However is isotropic media it is a scalar quantity havind same dimension as 'μ_0'.

5) Relative permeability: Relative permeability of the medium 'μ_r' is a dimensionless parameter given by the relation:

$$\mu_r = \frac{\mu}{\mu_0} \quad \text{..(1.5)}$$

Or $\quad \mu = \mu_r\mu_0 \quad \text{..(1.6)}$

From equation (1.4) and (1.6):

$$B = \mu_r\mu_0H \quad \text{..(1.7)}$$

Which when substituted in equation (1.3) gives

$$\mu_r = 1 + \chi \quad \text{..(1.8)}$$

For free space i.e. in absence of any material medium: **M** = 0, χ= 0, μ = μ$_0$ and μ$_r$ = 1. This gives

      B = μ$_0$H   ………………………………………………………….(1.9)

Types of Magnetism

Magnetism, at its root, arises from two sources:

1. Electric current.
2. Spin magnetic moments of elementary particles.

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1. Diamagnetism

2. Paramagnetism

3. Ferromagnetism

4. Antiferromagnetism

5. Ferrimagnetism

Diamagnetism

Diamagnetism is a very weak effect which appears in all materials and is the tendency of a material to oppose an applied magnetic field. Therefore diamagnetic materials are feebly repelled by a magnetic field. However, in a material with paramagnetic properties (that is, with a tendency to enhance an external magnetic field), the paramagnetic behavior dominates. Thus, despite its universal occurrence, diamagnetic behavior is observed only in a purely diamagnetic material. In a diamagnetic material, there are no unpaired electrons, so the intrinsic electron magnetic moments cannot produce any bulk effect. In these cases, the magnetization arises from the electrons' orbital motions, which can be understood classically as follows:

When a material is put in a magnetic field, the electrons circling the nucleus will experience, in addition to their Coulomb attraction to the nucleus, a Lorentz force from the magnetic field. Depending on which direction the electron is orbiting, this force may increase the centripetal force on the electrons, pulling them in towards the nucleus, or it may decrease the force, pulling them away from the nucleus. This effect systematically increases the orbital magnetic moments that were aligned opposite the field and decreases the ones aligned parallel to the field (in accordance with Lenz's law). This results in a small bulk magnetic moment, with an opposite direction to the applied field.

Paramagnetism

Paramagnetism is a very weak effect which appears in some materials and is the tendency of a material to reinforce an applied magnetic field. Therefore diamagnetic materials are feebly attracted by a magnetic field. In a paramagnetic material there are *unpaired electrons*; i.e., atomic or molecular orbitals with exactly one electron in them. While paired electrons are required by the Pauli exclusion principle to have their intrinsic ('spin') magnetic moments pointing in opposite directions, causing their magnetic fields to cancel out, an unpaired electron is free to align its magnetic moment in any direction. When an external magnetic field is applied, these magnetic moments will tend to align themselves in the same direction as the applied field, thus reinforcing it. Paramagnetism occurs in

i) Metals

ii) Atoms and molecules possessing odd number of electrons for e.g. Sodium atom, gaseous Nitric oxide.

iii) A few compounds having an even number of electrons e.g. oxygen molecule

iv) Free atoms and ions having a partially filled inner shell e.g. rare earth and actinide elements, ions of transition elements.
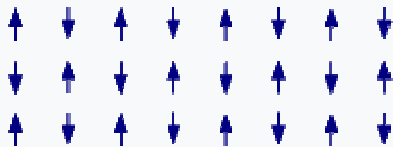
Ferromagnetism

A ferromagnet, like a paramagnetic substance, has unpaired electrons. However, in addition to the electrons' intrinsic magnetic moment's tendency to be parallel to an applied field, there is

also in these materials a tendency for these magnetic moments to orient parallel to each other to maintain a lowered-energy state. Thus, even in the absence of an applied field, the magnetic moments of the electrons in the material spontaneously line up parallel to one another. **Thus Ferromagnetism is a strong effect and is the tendency of a material to strongly reinforce an applied magnetic field. Therefore diamagnetic materials are strongly attracted by a magnetic field.**

Every ferromagnetic substance has its own individual temperature, called the Curie temperature, or Curie point, above which it loses its ferromagnetic properties. This is because the thermal tendency to disorder overwhelms the energy-lowering due to ferromagnetic order.

Ferromagnetism only occurs in a few substances; common ones are iron, nickel, cobalt, their alloys, and some alloys of rare-earth metals.
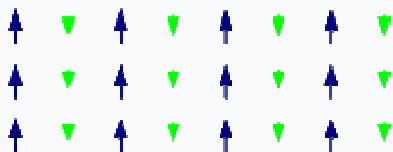
Antiferromagnetism[edit]

Antiferromagnetic ordering

In an antiferromagnet, unlike a ferromagnet, there is a tendency for the intrinsic magnetic moments of neighboring valence electrons to point in *opposite* directions. When all atoms are arranged in a substance so that each neighbor is anti-parallel, the substance is **antiferromagnetic**. **Antiferromagnets have a zero net magnetic moment, meaning that no field is produced by them.** Antiferromagnets are less common compared to the other types of behaviors and are mostly observed at low temperatures. In varying temperatures, antiferromagnets can be seen to exhibit diamagnetic and ferromagnetic properties.

In some materials, neighboring electrons prefer to point in opposite directions, but there is no geometrical arrangement in which *each* pair of neighbors is anti-aligned. This is called a spin glass and is an example of geometrical frustration.

Ferrimagnetism[edit]

Ferrimagnetic ordering

Like ferromagnetism, **ferrimagnets** retain their magnetization in the absence of a field. However, like antiferromagnets, neighboring pairs of electron spins tend to point in opposite directions. These two properties are not contradictory, because in the optimal geometrical arrangement, there is more magnetic moment from the sublattice of electrons that point in one direction, than from the sublattice that points in the opposite direction.

Most ferrites are ferrimagnetic. The first discovered magnetic substance, magnetite, is a ferrite and was originally believed to be a ferromagnet; Louis Néel disproved this, however, after discovering ferrimagnetism.

Classical Langevin theory of diamagnetism

Let an electron of a material be considered, revolving round the nucleus in circular orbit of radius 'r' with angular frequency 'ω_0' in absence of any superimposed magnetic field .

$$m\omega_0^2 r = \frac{Ze^2}{4\pi\varepsilon_0 r^2} \quad \text{...(1.10)}$$

or $\quad \omega_0 = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 mr^3}} \quad \text{.......................................(1.11)}$

Where 'Z' is the atomic number of the concerned atom, 'Ze' is the nuclear charge, 'e' is the charge of an electron, and ε_0 is the permittivity of free space.

A moving electron being equivalent to current, some magnetic flux is linked with such a current loop. When an external magnetic field is applied, the magnetic flux linked with this current loop tends to change. According to Lenz's law this change alters the current in the loop in such a way so as to oppose the change in flux. Consequently, the frequency of revolution, changes. If the electron be moving with a velocity ' $\vec{v}$' then in the presence of a magnetic field of magnetic induction '$\vec{B}$' the Lorentz force on it is

$$\vec{F} = e(\vec{v} \times \vec{B}) \quad \text{...(1.12)}$$

Assuming the magnetic field to be perpendicular to the electron orbit

$\quad F = evB = er\omega B \quad \text{...(1.13)}$ as $v = r\omega$, ω being the angular frequency of revolution in presence of magnetic field '$\vec{B}$' . The corresponding centripetal force on the electron is given by

$\quad m\omega^2 r = $ Attractive force on electron by nucleus + Lorentz force

or $\quad m\omega^2 r = \frac{Ze^2}{4\pi\varepsilon_0 r^2} - er\omega B = m\omega_0^2 r - er\omega B \quad \text{............................(1.14)}$

The (-) sign with the Lorentz is due to the fact that electron has negative charge. This gives

$$\omega^2 + \frac{eB}{m}\omega - \omega_0^2 = 0 \quad \text{.......................................(1.15)}$$

This is a quadratic equation in 'ω' whose solution is given by

$$\omega = \frac{1}{2}\left\{-\frac{eB}{m} \pm \sqrt{\left(\frac{eB}{m}\right)^2 + 4\omega_0^2}\right\} = \pm\sqrt{\left(\frac{eB}{2m}\right)^2 + \omega_0^2} - \frac{eB}{2m} \quad \text{............................(1.16)}$$

For $\dfrac{eB}{2m} \ll \omega_0$ we have

$$\omega = \pm\omega_0 - \frac{eB}{2m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.17)$$

This shows that in presence of a magnetic field of magnetic induction '$\vec{B}$' the angular frequency of revolution of the electron changes by a factor of $\dfrac{eB}{2m}$. The $\pm$ sign in equation (1.17) signifies that the electrons with orbital moments parallel to the field are slowed down, while those with moments, antiparallel to the field are speeded up. This result is called **Larmor Theorem.** The change in frequency produces an additional current given by:

$\qquad$ I = charge on electron $\times$ change in revolutions per unit time

Or $\qquad$ $I = (-e)\dfrac{1}{2\pi}\dfrac{eB}{2m} = -\dfrac{e^2B}{4\pi m} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.18)$ $\quad$ as frequency $= \dfrac{1}{2\pi}$ angular frequency

The magnetic moment of the current loop containing a single electron is given by

$$\mu_e = I \times \text{Area of the loop} = -\frac{e^2B}{4\pi m} \times \pi r^2 = -\frac{e^2B}{4m}r^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.19)$$

The negative sign denotes that the induced magnetic moment points in the direction opposite to the applied magnetic field.

If the magnetic field '$\vec{B}$' erpendicular to the electron orbit then 'r' in equation (1.19) has to be replaced by 'ρ' , the projection of 'r' on a plane perpendicular to the magnetic field. If an atom contains 'Z' electrons with their orbits randomly oriented, then the average total induced magnetic moment becomes :

$$\mu_a = -\frac{Ze^2\vec{B}}{4m}<\rho^2> \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.20)$$

If the field points along Z axis then : $<\rho^2> = <x^2> + <y^2>$ is the mean of the squares of the perpendicular distances of the electrons from the axis of the field. The mean square distances of the electrons from the nucleus is :

$$<r^2> = <x^2> + <y^2> + <z^2>$$

For a spherically symmetric charge distribution: $<x^2> = <y^2> = <z^2>$, which gives

$$<r^2> = 3<x^2> = \frac{3}{2}(<x^2> + <y^2>) = \frac{3}{2}<\rho^2> \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.21)$$

From (1.20) and (1.21) :

$$\mu_a = -\frac{Ze^2\vec{B}}{6m}<r^2> = -\frac{\mu_0 Ze^2\vec{H}}{6m}<r^2> \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.22)$$

Where 'μ_0' is the permeability of free space and 'H' is the magnetic field intensity. For a solid containing 'N' atoms per unit volume, each containing 'Z' electrons, the expression for magnetic moment per unit volume is :
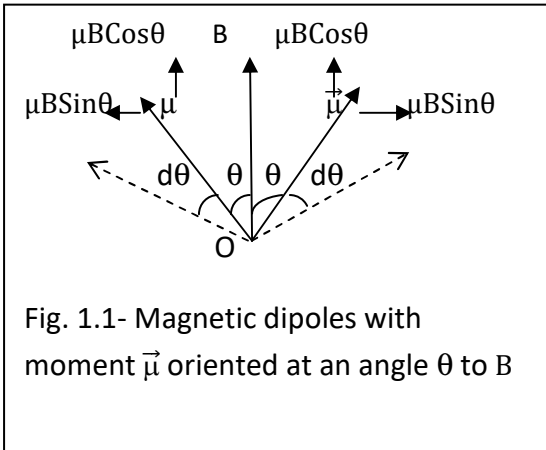
$$\vec{M} = -\frac{\mu_0 ZNe^2\vec{H}}{6m}<r^2> \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1.23)$$

And the diamagnetic susceptibility is :

$$\chi = = \frac{M}{H} = -\frac{\mu_0 ZNe^2}{6m}$$ …………………………………………….(1.24)

This is Classical Langevin result which shows that the diamagnetic susceptibility is independent of temperature.

Classical Langevin theory of Paramagnetism.

Fig. 1.1- Magnetic dipoles with moment $\vec{\mu}$ oriented at an angle θ to B

Let a paramagnetic gas be considered with 'N' atoms per unit volume, each having a permanent magnetic moment '$\vec{\mu}$', The mutual interaction between the magnetic dipoles are assumed to be negligible. In the presence of a magnetic induction '$\vec{B}$' these dipoles tend to orient themselves in the direction of the applied field in order to minimize their energy. However, due to thermal energy at ordinary temperature an inherent randomness tend to resist such an alignment of dipoles. So instead of being perfectly aligned, the dipoles, at thermal equilibrium orient themselves at angle 'θ' to '$\vec{B}$' as shown in fig. 1.1. The potential energy of each dipole in this position is given by:

$$E_\theta = -\vec{\mu}.\vec{B} = -\mu B Cos\theta$$ ……………………………..(1.25)

If 'n' be the number of magnetic dipoles oriented at angle 'θ' to '$\vec{B}$' then according to Maxwell Boltzmann distribution Law

$$n \, \alpha \, e^{\left(-\frac{E_\theta}{KT}\right)}$$

$$n \, \alpha \, e^{\left(\frac{\mu B Cos\theta}{KT}\right)}$$

Also the probability for a magnetic dipole to make an angle 'θ' to '$\theta+d\theta$' with '$\vec{B}$' is given by:

$$dP_\theta \, \alpha \, e^{\left(\frac{\mu B Cos\theta}{KT}\right)} d\omega$$

Where '$d\omega$' is the solid angle lying between the two hollow cones of semivertical angles 'θ' to '$\theta+d\theta$'.Hence the number of dipoles having axis within the solid angle '$d\omega$' is given by

$$dn \, \alpha \, e^{\left(\frac{\mu B Cos\theta}{KT}\right)} d\omega$$

or $\quad dn = ke^{\left(\frac{\mu B Cos\theta}{KT}\right)} d\omega$

or $\quad dn = ke^{\left(\frac{\mu B Cos\theta}{KT}\right)} Sin\theta d\theta$ …………………………………(1.26)

where 'k' is the proportionality constant. Each of these dipoles contributes a component of $\mu Cos\theta$ to the magnetization, whereas the components perpendicular to the field direction cancel each other as shown in fig. (1.1). hence the average component of the magnetic moment of all the atoms along the field direction per unit volume i.e the magnetization is

$$M = N\mu < Cos\theta >$$

$$= N\frac{\int_0^\pi \mu Cos\theta dn}{\int_0^\pi dn} = N\mu \frac{k\int_0^\pi Cos\theta Sin\theta e^{\left(\frac{\mu BCos\theta}{KT}\right)}d\theta}{k\int_0^\pi e^{\left(\frac{\mu BCos\theta}{KT}\right)}Sin\theta d\theta} = N\mu \frac{\int_0^\pi Cos\theta Sin\theta e^{\left(\frac{\mu BCos\theta}{KT}\right)}d\theta}{\int_0^\pi e^{\left(\frac{\mu BCos\theta}{KT}\right)}Sin\theta d\theta} \quad \ldots\ldots(1.27)$$

Let $\frac{\mu B}{KT} = x$ and $Cos\theta = y$

$\therefore -Sin\theta d\theta = dy$

This gives: $M = N\mu \frac{\int_1^{-1} y e^{(xy)}dy}{\int_1^{-1} e^{(xy)}dy} = N\mu \frac{\left[y\int e^{(xy)}dy - \int\{\int e^{(xy)}dy\}dy\right]_1^{-1}}{\left(e^{\frac{xy}{y}}\right)_1^{-1}} = N\mu \frac{\left(\frac{ye^{xy}}{x} - \frac{1}{x}\int e^{xy}dy\right)_1^{-1}}{\frac{1}{x}(e^{xy})_1^{-1}}$

$$= N\mu \frac{\left(ye^{xy} - \frac{e^{xy}}{x}\right)_1^{-1}}{e^{-x} - e^x} = N\mu \frac{-e^{-x} - e^x - \frac{1}{x}(e^{-x} - e^x)}{e^{-x} - e^x} = N\mu\left\{-\left(\frac{e^{-x} + e^x}{e^{-x} - e^x}\right) - \frac{1}{x}\right\}$$

$$= N\mu\left\{\left(\frac{e^{-x} + e^x}{e^x - e^{-x}}\right) - \frac{1}{x}\right\}$$

Or $\quad M = N\mu\left\{Coth(x) - \frac{1}{x}\right\} = N\mu L(x)\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(1.28)$
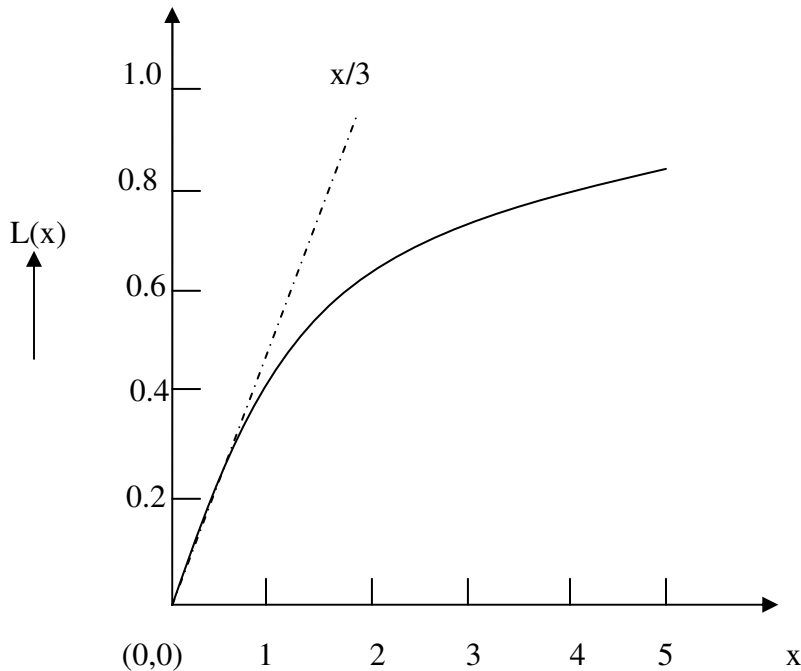
Where L(x) is called the **Langevin function.**

Fig. 1.2: Variation of L(x) with 'x'

The figure above shows the variation of L(x) with 'x'. The graph exhibits that for 'x' << 1 i.e. at normal field strengths and ordinary temperature the curve is almost linear coincides with the tangent to the curve at the origin which is $\frac{x}{3}$. Thus we have :

At $x \to 0$ $L(x) = \dfrac{x}{3} = \dfrac{\mu B}{3KT}$ ……………………………………….(1.29)

Substituting equation (1.29) in eqn. (1.28)

$$M = \frac{N\mu^2 B}{3KT} = \frac{N\mu^2 \mu_0 H}{3KT} \qquad \text{…………………………(1.30)}$$

The paramagnetic susceptibility is hence given by

$$\boxed{\chi = \frac{M}{H} = \frac{\mu_0 N\mu^2}{3KT}} \qquad \text{……………………………..(1.31)}$$

This shows that the paramagnetic susceptibility is inversely proportional to temperature. Substituting ' $\dfrac{\mu_0 N\mu^2}{3K}$ ' as a constant 'C' it can be written:

$$\boxed{\chi = \frac{C}{T}} \qquad \text{………………………………………(1.33)}$$

The expression (1.33) is called the **Curie Law** and the constant $C = \dfrac{\mu_0 N\mu^2}{3K}$ is called **Curie Constant. Curie law holds good only for 'x' $\to$ 0, i.e. $\dfrac{\mu B}{KT} \to 0$ or μB << KT**

For large values of 'x' i.e for high values of 'B' which makes μB >> KT, L(x) approaches unity and equation (1.28) becomes:

$$M = N\mu = M_S \qquad \text{……………………………(1.34)}$$

Thus a saturation of magnetization is reached which corresponds to the complete alignment of all the magnetic dipoles in the field direction. **The quantity M_S is called the Saturation Magnetization. Consequently magnetic susceptibility also reaches a saturation.**