# RADIOACTIVITY

Radioactivity is the phenomenon exhibited by the nuclei of an atom as a result of nuclear instability. **Radioactivity** is the property possessed by some elements (such as uranium) or isotopes (such as carbon 14) of spontaneously emitting energetic particles (such as electrons or alpha particles) by the disintegration of their atomic nuclei. **Radioactive decay** (also known as **nuclear decay**, **radioactivity**, **radioactive disintegration** or **nuclear disintegration**) is the process by which an unstable atomic nucleus loses energy by radiation. A material containing unstable nuclei is considered **radioactive**. Except for gamma decay or internal conversion from a nuclear excited state, the decay is a nuclear transmutation resulting in a daughter containing a different number of protons or neutrons (or both). When the number of protons changes, an atom of a different chemical element is created.

A — Radioactive disintegration — B

When an atom A disintegrates to another atom B, the decaying nucleus A is called the *parent radionuclide* (or *parent radioisotope*), and the process produces at least one *daughter nuclide* B.

## Units of Radioactivity

The <u>International System of Units</u> (SI) unit of radioactive activity is the <u>becquerel</u> (Bq), named in honor of the scientist <u>Henri Becquerel</u>. One Bq is defined as one transformation (or decay or disintegration) per second.

An older unit of radioactivity is the <u>curie</u>, (Ci), which was originally defined as "the quantity or mass of <u>radium emanation</u> in <u>equilibrium</u> with one gram of <u>radium</u> (element)".<sup>[17]</sup> Today, the curie is defined as  $3.7 \times 10^{10}$  disintegrations per second, so that 1 <u>curie</u> (Ci) =  $3.7 \times 10^{10}$  Bq. For radiological protection purposes,

## Stability of nucleus

Nuclear **stability** means that the concerned nucleus does not spontaneously emit any kind of radiation. On the other hand, if the **nucleus** is unstable, it has the tendency of emitting some kind of radiation, which makes it radioactive. An atom is **stable** if the forces among the particles that constitute the **nucleus** are balanced. An atom is unstable (radioactive) if these forces are unbalanced, i.e. if the **nucleus** has an excess of internal energy. Instability of an atom's **nucleus** may result from an excess of either neutrons or protons.

The **ratio** of neutrons to protons (n/p) is a successful way in predicting nuclear **stability**. This **ratio** is close to 1 for atoms of elements with low atomic numbers (of less than about 20 protons). The **n/p ratio** steadily increases as the atomic number increases past element 20 (calcium) to about element 84 (polonium).



#### Fig. 1.1.

At close distances, a strong nuclear force exists between nucleons. This attractive force comes from the neutrons. More protons in the nucleus need more neutrons to bind the nucleus together. The graph above is a plot of the number of neutrons versus the number of protons in various stable <u>isotopes</u>. The stable nuclei are in the pink band known as the **belt of stability**. They have a neutron/proton ratio between 1:1 and 1.5.

As the nucleus gets bigger, the electrostatic repulsions between the protons get weaker. The nuclear strong force is about 100 times as strong as the electrostatic repulsions, but it operates over only short distances. After a certain size, the strong force is not able to hold the nucleons together. Adding extra neutrons increases the space between the protons. This decreases their repulsions but, if there are too many neutrons, the nucleus is again out of balance and decays.

If a heavy nucleus were to split into two fragments, each fragment would form a nucleus that would have approximately the same neutron-to-proton ratio as the heavy nucleus. This high neutron-to-proton ratio places the fragments below and to the right of the stability curve displayed by the Figure (1.1).

# **Fundamental Laws of Radioactivity**

1. Soddy and Fajan's displacement law: Algebraic sum of the atomic number of the parent nuclei before disintegration should be equal to the algebraic sum of the atomic number of the daughter nuclei after disintegration. Also algebraic sum of the mass number of the parent nuclei before disintegration should be equal to the algebraic sum of the mass number of the daughter nuclei after disintegration.

$$X_{Z_{1}}^{A_{1}} \longrightarrow Radioactive decay \longrightarrow Y_{Z_{2}}^{A_{2}} + B_{Z_{3}}^{A_{3}} \} A = mass number; Z = atomic number$$
  
Then:  $A_{1} = A_{2} + A_{3}$   
 $Z_{1} = Z_{2} + Z_{3}$ 

2. Law of Radioactive disintegration: For an element undergoing radioactive disintegration, at any instant, the rate of radioactive disintegration is directly proportional to the number of nuclei of the radioactive element present at that instant. If 'N' be the number of nuclei of the radioactive element present at any instant 't', then according to the Law of Radioactive decay:

Or  $\frac{dN}{dt} \alpha - N$  $\frac{dN}{dt} = -\lambda N$  .....(1.2)

Where ' $\lambda$ ' is the proportionality constant called **decay constant**. The negative sign denotes that the number of undisintigrated nuclei decreases with time

3. Law of successive transformations: In general one radioactive parent element decays to another daughter element which is also radioactive. The process of radioactive decay does not cease with the formation of the daughter, but extends over many generations, until a stable end product is reached.

# Result obtained from the law of radioactive disintegration

As indicated by equation (1.2):

$$\frac{dN}{dt} = -\lambda N$$

Or 
$$\frac{dN}{N} = -\lambda dt$$

 $\text{Or} \qquad \int_{N_0}^N \frac{dN}{N} \; = \; -\lambda \int_0^t dt \qquad \text{where $N_0$ is the initial number of radioactive nuclei}.$ 

Integrating we get:

$$ln\left(\frac{N}{N_{0}}\right) = -\lambda t$$
  
Or  $\frac{N}{N_{0}} = e^{-\lambda t}$   
Or  $N = N_{0} e^{-\lambda t}$  .....(1.2)

Equation (1.2) gives the number of radioactive nuclei present at time 't' after the dissintigration starts. The graph of  $\frac{N}{N_0}$  vs. time is given below.



Fig. 1.2. The graph of  $\frac{N}{N_0}$  vs. time

# Half life of a radioactive element $\left(T_{\frac{1}{2}}\right)$

Half life of a radioactive element ' $T_{\frac{1}{2}}$ ' is defined as the time in which it disintegrates to half of its initial amount.



### Mean life (T<sub>m</sub>)

**Mean life**, in <u>radioactivity</u>, is the average lifetime of all the nuclei of a particular unstable atomic species. This time interval may be thought of as the sum of the lifetimes of all the individual unstable nuclei in a sample, divided by the total number of unstable nuclei present.

Let dN be the number of nuclei which disintigrates within time t to t+dt. So lifetime of each of dN nuclei is on an average = 't' . Hence the sum of lifetime of each dN nuclei is = tdN. So considering all the N<sub>0</sub> nuclei the sum of lifetime of all N<sub>0</sub> nuclei is

$$\mathsf{T}_{\mathsf{total}} = \int_{N_0}^0 t dN$$

From equation (1.2)  $N = N_0 e^{-\lambda t}$ 

Or  $-dN = \lambda N_0 e^{-\lambda t} dt$ , -dN indicates that N decreases as the nuclei disintigrates

So  $T_{total} = \lambda N_0 \int_0^\infty t e^{-\lambda t} dt$  the total time taken for all nuclei to disintegrate is  $\infty$ 

Substituting in equation (1.4):

$$T_{m} = \frac{1}{N_{0}} \frac{N_{0}}{\lambda} = \frac{1}{\lambda}$$
  
So Mean life = 
$$T_{m} = \frac{1}{\lambda}$$
 .....(1.6)

## Relation between half life and mean life.

 $T_{\frac{1}{2}} = 0.693 T_{m}$  (1.7)

Half life =  $0.693 \times$  Mean life

# Result obtained from the law of successive transformations

A successive transformations be considered from  $1 \rightarrow 2 \rightarrow 3$  with decay constants  $\lambda_1$ ,  $\lambda_2$ , Also let N<sub>10</sub>, be the initial number of radioactive substances 1 present. Befor disintegration starts no atom of element (2) is present. (2) is subsequently created after disintegration starts. Now it is required to find the number of corresponding atoms N<sub>1t</sub>, N<sub>2t</sub>, present after time 't'. From the law of radioactive decay:

as  $N_2$  is produced at a rate  $\lambda_1 N_1$  and decays at a rate  $\lambda_2 N_2$ 

From equation (1.3) N<sub>1</sub> = N<sub>10</sub> $e^{-\lambda_1 t}$ 

Inserting this expression of N<sub>1</sub> in equation (1.4)

$$\frac{dN_2}{dt} = \lambda_1 N_{10} e^{-\lambda_1 t} - \lambda_2 N_2$$

 $\label{eq:or} \text{Or} \qquad \frac{dN_2}{dt} + \lambda_2 N_2 \, = \, \lambda_1 \, N_{10} \, e^{-\lambda_1 t}$ 

Multiplying throughout by  $e^{\lambda_2 t}$  we get

$$e^{\lambda_2 t} \frac{dN_2}{dt} + \lambda_2 N_2 e^{\lambda_2 t} = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

$$\text{Or} \qquad \frac{d(N_2 e^{\lambda_2 t})}{dt} = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}$$

 $\label{eq:constraint} \text{Or} \qquad d(N_2 e^{\lambda_2 t}) = \lambda_1 N_{10} e^{(\lambda_2 - \lambda_1) t} \text{d} t$ 

Integrating

$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1} + C$$
, where C is the constant of integration

At t = 0,  $N_2 = 0$ , which gives

$$C = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}$$

Or 
$$N_2 e^{\lambda_2 t} = \frac{\lambda_1 N_{10} e^{(\lambda_2 - \lambda_1)t}}{\lambda_2 - \lambda_1} + \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1}$$

Or 
$$N_2 = \frac{\lambda_1 N_{10} e^{-\lambda_1 t}}{\lambda_2 - \lambda_1} + \frac{\lambda_1 N_{10} e^{-\lambda_2 t}}{\lambda_2 - \lambda_1}$$

Or 
$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$
 .....(1.5)

This gives the number of elements of (2) present in a time 't' after disintegration starts.

The treatment may be extended to a chain of any number of successive transformations  $1\rightarrow 2\rightarrow 3\rightarrow 4\rightarrow 5$  ...... But the mathematical treatment becomes quite tedious as we will have to deal with a number of equations and constants.

## Radioactive equilibrium

When an isotope of an element is unstable due to its ratio of neutrons to protons, it gives off either particles, such as neutrons, protons or electrons, or an electromagnetic wave, and a new isotope is formed. The **parent isotope** decays into the **daughter isotope**. Sometimes the daughter isotope is unstable as well, and it begins to decay as shown below.

$$1 \rightarrow 2 \rightarrow 3$$

Radioactive equilibrium is attained when the decay rate of a daughter radioactive isotope equals the production rate of that isotope by another source. In other words radioactive equilibrium is а condition, in which, а radioactive species and its successive radioactive products have attained such relative proportions that they all disintegrate at the same numerical rate and therefore maintain their proportions constant. Equation (1.5) shows that N<sub>2</sub> is zero both at t = 0 and t =  $\infty$ . Hence the activity element (2) must pass through a maximum value at some intermediate value of time viz. ' $t_{m^\prime}$  . At the maxima  $\frac{dN_2}{dt} = 0$ . Differentiating equation (1.5) and equating it to zero:

$$\frac{dN_2}{dt} = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left( -\lambda_1 e^{-\lambda_1 t} + \lambda_2 e^{-\lambda_2 t} \right) = 0 \text{ at } t = t_m$$

0

$$Or \qquad -\lambda_1 e^{-\lambda_1 t_m} + \lambda_2 e^{-\lambda_2 t_m} =$$

$$\mbox{Or} \qquad \lambda_1 e^{-\,\lambda_1 t_m} = \,\lambda_2 e^{-\,\lambda_2 t_m}$$

Or 
$$e^{(\lambda_2 - \lambda_1)t_m} = \frac{\lambda_2}{\lambda_1}$$

Or 
$$t_m = \frac{\ln(\frac{\lambda_2}{\lambda_1})}{\lambda_2 - \lambda_1}$$
 .....(1.6)

## Types of radioactive equilibrium

## 1. Secular equilibrium:

Let a parent radioactive specimen be A. It disintegrates to a daughter element B which also has an unstable nucleus and hence in turn disintegrates to another element C

1 — Disintigration 
$$\rightarrow$$
 Disintigration  $\rightarrow$ 

 $\lambda_1 \! \rightarrow$  Decay constant of A ;  $\, \lambda_2 \, \rightarrow$  decay constant of B

Secular equilibrium can only occur in a radioactive decay chain if daughter radionuclide 2 has a much shorter half-life than the parent radionuclide 1. If T<sub>1</sub> and T<sub>2</sub>be the half life of parent and daughter (1) and (2) respectively and the corresponding decay constants are  $\lambda_1$  and  $\lambda_2$ then  $T_2 \ll T_1$ . Consequently:  $\lambda_1 \ll \lambda_2$ .

Referring to equation (1.5): 
$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left( e^{-\lambda_1 t} - e^{-\lambda_2 t} \right)$$

: Compared to  $\lambda_2$ ,  $\lambda_1 \rightarrow 0$ . Hence  $\lambda_2 - \lambda_1 \approx \lambda_2$  and  $e^{-\lambda_1 t} = 1$ . This gives:

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2} \left( 1 - e^{-\lambda_2 t} \right) \qquad (1.10)$$

For a time 't' >> T<sub>2</sub>,  $e^{-\lambda_2 t} \rightarrow 0$  and equation (1.10) reduces to

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2}$$
 .....(1.11)

In such a situation, the decay rate of A, and hence the production rate of 2, is approximately constant, because the half-life of 1 is very long compared to the time scales being considered. The quantity of radionuclide 2 builds up until the number of 2 atoms decaying per unit time becomes equal to the number being produced per unit time; the quantity of radionuclide 2 then reaches a constant, equilibrium value. Assuming the initial concentration of radionuclide 2 is zero, full equilibrium usually takes several half-lives of radionuclide B to establish. If N<sub>1</sub> and N<sub>2</sub> be the number of nuclei of type A and B present at any instant, then: Activity of (1)

$$A_1 = \frac{dN_1}{dt} = -\lambda_1 N_1$$
 = rate of disintegration of 1 = rate of production of 2

 $-\lambda_2 N_2$  = rate of disintegration of 2

Net time rate of change of the number of atoms of radionuclide 2 i.e the net activity

$$A_2 = \frac{dN_2}{dt} = -\lambda_1 N_1 - (-\lambda_2 N_2) = \lambda_2 N_2 - \lambda_1 N_1$$

When secular equilibrium is reached the amount of B should remain constant.

$$\therefore \qquad A_2 = \frac{dN_2}{dt} = 0$$

 $\lambda_2 N_2 - \lambda_1 N_1 = 0$ Or

 $\lambda_1 N_1 = \lambda_1 N_1 \qquad (1.12)$ Or

If we have a series of successive disintigrations:  $1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow \dots$  Then at secular equilibrium we have

 $\lambda_1 N_1 = \lambda_2 N_2 = \lambda_3 N_3 = \lambda_4 N_4$ An example of this is the decay of Ra<sup>226</sup> with half life 1620 years to Rn<sup>226</sup> of half life 4.8 days.



#### **Transient equilibrium**

Transient equilibrium occurs if the daughter nucleus has somewhat shorter half life as compared to its parents. If,  $T_1$  and  $T_2$  be the half life of parent and daughter (1) and (2) respectively, then  $T_2 < T_1$  and consequently  $\lambda_1 < \lambda_2$ . After a sufficiently long time compared to the half life of (2),  $e^{-\lambda_2 t}$  becomes small enough to be negligible. When that time span has passed, a state of equilibrium called transient equilibrium will be reached. Ignoring  $e^{-\lambda_2 t}$  equation (1.5) becomes

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left( e^{-\lambda_1 t} \right) \qquad (1.14)$$

Substituting,  $N_{10}e^{-\lambda_1 t} = N_1$  in equation (1.14)

Thus at equilibrium the ratio of the two activities at large 't' compared to T<sub>2</sub>

$$\frac{\mathbf{A_1}}{\mathbf{A_2}} = \frac{\frac{\mathrm{dN_1}}{\mathrm{dt}}}{\frac{\mathrm{dN_2}}{\mathrm{dt}}} = \frac{\lambda_1 \mathbf{N_1}}{\lambda_2 \mathbf{N_2}} = \frac{\lambda_1 (\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2} = \frac{(\lambda_2 - \lambda_1)}{\lambda_2}$$

As half life is inversely proportional to decay constant

$$\frac{(\lambda_2 - \lambda_1)}{\lambda_2} = \frac{\frac{1}{T_2} - \frac{1}{T_1}}{\frac{1}{T_1}} = \frac{T_1 - T_2}{T_1}$$

So 
$$\frac{A_1}{A_2} = \frac{(\lambda_2 - \lambda_1)}{\lambda_2} = 1 - \frac{\lambda_1}{\lambda_2} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$
 ....(1.16)

Thus  $\frac{A_1}{A_2}$  may have any value between 0 and 1 depending upon the ratio  $\frac{\lambda_1}{\lambda_2}$ . The build up of Ra<sup>223</sup> with a half life of 11.2 days from Th<sup>227</sup> with a half life of 18.9 days is an example of this case.



#### Non equilibrium

1...

Equilibrium is never reached if the daughter nucleus has a longer half life as compared to its **parent.** If,  $T_1$  and  $T_2$  be the half life of parent and daughter (1) and (2) respectively, then  $T_2 > T_1$ and consequently  $\lambda_1 > \lambda_2$ . In this case the ratio of the activities of the daughter to parent is:

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In the extreme case when  $\lambda_1 >> \lambda_2$  then  $e^{-\lambda_1 t}$  is  $\ll e^{-\lambda_2 t}$ . This gives for 't'  $>> T_1$ 

$$N_2 = \frac{\lambda_1 N_{10}}{\lambda_2 - \lambda_1} \left( - e^{-\lambda_2 t} \right)$$

Or

Also as when  $\lambda_1 >> \lambda_2$ , hence  $\lambda_2 - \lambda_1 \approx -\lambda_1$ . So the activity of (2) becomes:

An example of this is the decay of  $Bi^{210}$  with half life 5 days to  $Po^{210}$  of half life 138 days. The activity of polonium is at a maximum after 25 days when only 3% of the initial number of bismuth is left.



## Daughter and parent nuclei are of nearly equal half life.

If the half lives of parent (T<sub>1</sub>) and daughter (T<sub>3</sub>) are nearly equal then  $\lambda_1 \approx \lambda_2$ . Let

$$T_{1} = T_{2}(1 + \varepsilon) \qquad (1.8) \quad \text{where } \varepsilon << 1.$$
Now
$$\frac{A_{2}}{A_{1}} = \frac{\lambda_{2} \left\{ e^{(\lambda_{1} - \lambda_{2})t} - 1 \right\}}{(\lambda_{1} - \lambda_{2})} \qquad (1.8) \quad \text{from equation (1.17)}$$

$$= \frac{\frac{1}{T_{2}} \left\{ e^{\lambda_{2} \left( \frac{T_{2}}{T_{1}} - 1 \right)t} - 1 \right\}}{\left( \frac{1}{T_{1}} - \frac{1}{T_{2}} \right)} \qquad (1.8) \quad \text{as T is inversely proportional to } \lambda$$

$$= \frac{T_{1}T_{2} \left\{ e^{\lambda_{2} \left( \frac{T_{2}}{T_{1}} - 1 \right)t} - 1 \right\}}{T_{2}(T_{2} - T_{1})} \qquad (1.8)$$

$$= \frac{(1 + \varepsilon) \left\{ 1 - e^{-\lambda_{2} \left( \frac{\varepsilon}{1 + \varepsilon} \right)t} \right\}}{\varepsilon} \qquad \text{using equation (1.8)}$$

$$= \frac{(1 + \varepsilon) \left\{ 1 - 1 + \lambda_{2} \left( \frac{\varepsilon}{1 + \varepsilon} \right)t \right\}}{\varepsilon} \qquad \text{Assuming } t << \frac{1}{\varepsilon \lambda_{2}} \text{ and expanding binomially}$$

$$= \frac{(1 + \varepsilon) \left\{ \lambda_{2} \left( \frac{\varepsilon}{1 + \varepsilon} \right)t \right\}}{\varepsilon}$$

So 
$$\frac{A_2}{A_1} = \lambda_2 t$$

So it is seen that the ratio of activities of daughter to parent increases linearly with time as long as  $t \ll \frac{1}{\varepsilon \lambda_2}$ . An example of this is the disintegration of Pb<sup>200</sup> with half life 21 hours to Thallium-200 of half life 26.4 hours.

# **Radioactive series**

Radioactive series (known also a **radioactive** cascades) as are three naturally occurring radioactive decay chains and one artificial radioactive decay chain of unstable heavy atomic nuclei that decay through a sequence of alpha and beta decays until a stable nucleus is achieved. Most radioisotopes do not decay directly to a stable state and all isotopes within the series decay in the same way. In physics of nuclear decays, the disintegrating nucleus is usually referred to as the parent nucleus and the nucleus remaining after the event as the daughter nucleus. Since alpha decay represents the disintegration of a parent nucleus to a daughter through the emission of the nucleus of a helium atom (which contains four nucleons), there are only four decay series. Within each series, therefore, the mass number of the members may be expressed as four times an appropriate integer (n) plus the constant for that series. As a result, the thorium series is known as the 4n series, the neptunium series as the 4n + 1 series, the uranium series as the 4n + 2 series and the actinium series as the Three of the sets are called natural or classical series. The fourth set, the neptunium series, is headed by neptunium-237. Its members are produced artificially by nuclear reactions and do not occur naturally.

- the thorium series (4n series),
- the uranium series (4n+2 series),
- the actinium series (4n+3 series),
- the neptunium series (4n+1 series).

The classical series are headed by <u>primordial unstable nuclei</u>. Primordial nuclides are nuclides found on the Earth that have existed in their current form since before Earth was formed. The previous four series consist of the radioisotopes, that are the descendants of four heavy nuclei with long and very long half-lives:

- the thorium series with thorium-232 (with a half-life of 14.0 billion years),
- the uranium series with uranium-238 (which lives for 4.47 billion years),
- the actinium series with uranium-235 (with a half-life of 0.7 billion years).
- the neptunium series with neptunium-237 (with a half-life of 2 million years).

Any such decay chain is only stopped by the formation of a stable nucleus. This occurs at the fourteenth generation of the uranium 238 family, when lead 206 is finally produced. The two other families, those formed from uranium 235 and thorium 232, end respectively with the creation of lead 207 and lead 208, two other stable isotopes of lead.