ALPHA DECAY

Alpha particles are composite **particles** consisting of two protons and two neutrons tightly bound together (Figure 1). They are in fact a helium nucleus i.e. a helium atom from which two electrons have been removed. They have a charge of +2e, e being the magnitude of charge on an electron. Symbol of alpha particles is $_2He^4$.

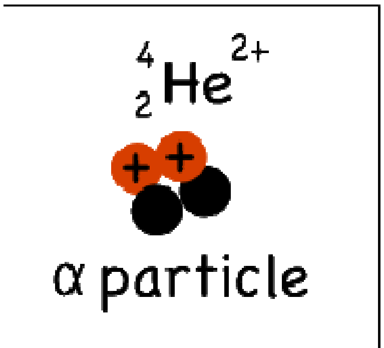


Fig. 2.1. Alpha particle

Alpha decay occurs in very heavy elements like uranium, thorium, and radium. They are called parent nucleus and they are basically unstable. Because the nuclei of these atoms have a lot more neutrons in their nuclei than protons, that is, they have too large a proton to neutron ratio, which makes these elements neutron-rich. This richness makes alpha decay possible. Thus, emitting its two protons and two neutrons in the form of an **alpha particle** and a forming of new daughter nucleus and attains a very stable configuration. **Alpha decay can be described like this:**

- The nucleus of these nuclei (parent nucleus) rich atoms splits into two parts.
- The alpha particle goes zooming off into space.
- The nucleus left behind (daughter nucleus) has its atomic number reduced by 2 and its mass number reduced by 4.

If $_ZX^A$ be a radioactive nucleus which emits an alpha particle producing a daughter nucleus Y, then the decay equation is:

$$_ZX^A = {}_{Z-2}Y^{A-4} + {}_2He^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2.1)$$

Some Alpha Decay Examples

$_{93}Np^{237} \rightarrow {}_{91}Pa^{233} + 2He^4$
$_{78}Pt^{175} \rightarrow {}_{76}Os^{171} + {}_2He^4$

Energy equation of alpha decay

An α-decay equation is represented by:

$$_{Z}P^{A} = {}_{Z-2}D^{A-4} + {}_{2}He^{4}$$

According to mass energy equivalence

$$M_{P}c^{2} = M_{D}c^{2} + M_{\alpha}c^{2} + E_{Y} + E_{\alpha} \quad \text{...(2.2)}$$

Where: M_P = mass of parent atom

M_Pc^2 = corresponding rest mass energy of parent atom

M_D = mass of daughter atom

MDc^2 = corresponding rest mass energy of daughter atom

M_α = mass of α particle

$M_\alpha c^2$ = corresponding rest mass energy of α particle

E_Y = kinetic energy imparted to the daughter nucleus

E_α = kinetic energy with which the α particle is ejected

Theory of alpha emission : The nuclear force holding an atomic nucleus together is very strong, in general much stronger than the repulsive electromagnetic forces between the protons. However, the nuclear force is also short range, dropping quickly in strength beyond about 1 femtometre, while the electromagnetic force has unlimited range. The strength of the attractive nuclear force keeping a nucleus together is thus proportional to the number of nucleons, but the total disruptive electromagnetic force trying to break the nucleus apart is roughly proportional to the square of its atomic number. A nucleus with 210 or more nucleons is so large that the strong nuclear force holding it together can just barely counterbalance the electromagnetic repulsion between the protons it contains. Alpha decay occurs in such nuclei as a means of increasing stability by reducing size.

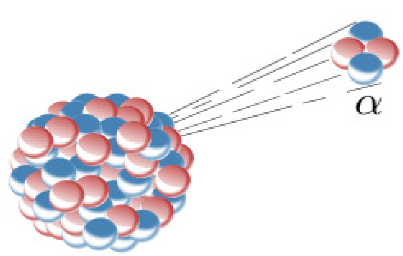


Fig. 2.2 Visual representation of alpha decay

Potential energy of alpha particles

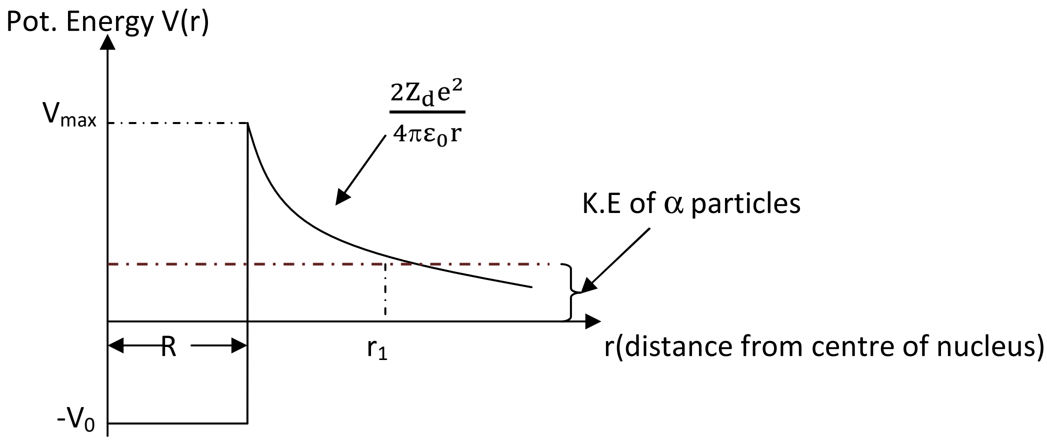


Fig. 2.3. Potential energy curve with nuclear radius = 10 fermi.

Taking into account the short ranged nuclear force and the Coulomb repulsive force the potential energy V(r) of an alpha particle at a distance 'r' from the centre of the nucleus of radius 'R' is given by

$$V(r) = \frac{2(Z_P - 2)e^2}{4\pi\varepsilon_0 r} = \frac{2Z_d e^2}{4\pi\varepsilon_0 r} \quad \text{for } r > R \quad \text{...(2.3)}$$

Where Z_P = atomic number of parent

$Z_P - 2 = Z_d$ = atomic number of daughter.

The variation of potential energy V(r) with 'r', the distance from the centre of the nuckeus of alpha emitter is shown in Fig.(1.3)

The maximum height of the potential barrier, which occurs at : r = R is

$$V_{max} = \frac{2Z_d e^2}{4\pi\varepsilon_0 R} \quad \text{......................................(2.4)}$$

For U^{238} the height of the potential barrier for an alpha particle is ≈ 26MeV.

Gamow theory of quantum mechanical approach, to alpha decay based on tunneling effect tunneling effect.

The theory of alpha decay was developed by Gamow in 1928 in collaboration with Gurney and Condon. The **assumptions on which this theory is based are:**

1. Since an alpha particle is emitted from a heavy nucleus as a discrete entity hence it can be inferred that it exists as a separate entity within the heavy nucleus.

2. The alpha particle is in constant motion within the nucleus under the effect of a surrounding potential barrier. It bounces back and forth from the barrier walls.

3 In each collision with the barrier the alpha particle has a definite probability that the particle will leak through the barrier. In fact the alpha particle hits the barrier wall repeatedly till conditions favourable for its penetration or leakage through the wall is achieved.

Let $\nu \rightarrow$ frequency with which an alpha particle collides with the potential barrier wall in order to escape from the nucleus.

$T \rightarrow$ Transmission probability in each collision.

$\lambda \rightarrow$ Radioactive decay constant of the heavy nucleus which emits the alpha particles = decay probability per unit time.

$$\lambda = \nu T \quad \text{...(2.5)}$$

Assuming that at a particular instant of time only one alpha particle exists as a discrete entity within the nucleus and that it moves back and forth along the nuclear diameter 'R':

$$\nu = \frac{v}{2R} \quad \text{...(2.6)}$$

where 'v' is the velocity of the alpha particle when it eventually leaves the nucleus. From equation (2.5) and (2.6)

$$\lambda = \frac{vT}{2R} \quad \text{...(2.7)}$$

The value of transmission probability of an α particle of kinetic energy E_α for a potential barrier of height V_0 and width 'a' is given by:

$$T = e^{-k_2 a} \quad \text{.....................................(2.8)}$$

Where $k_2 = \dfrac{\sqrt{2M_\alpha(V_0 - E_\alpha)}}{\hbar}$ ....(2.9)

But an α particle within the nucleus has to overcome a potential barrier of varying height as shown in Fig. (2.3)

Let r_1 the distance from the centre of the nucleus at which r = E_α. The potential is not constant within the region $r_1 > r > R$. Let this region be divided in a series of small sections each of width 'dr'. The probability of transition through a small portion from $r \rightarrow r+dr$ is given by:

$$T = e^{-2k_2 r dr} \text{...(2.10)}$$

Where $k_2(r) = \frac{\sqrt{2M_\alpha\{V(r) - E_\alpha\}}}{\hbar}$ ……………………………………(2.11)

$$V(r) = V(r) = \frac{2Z_d e^2}{4\pi\varepsilon_0 r}$$

So the sum of probability of individual probabilities of all small sections from r varying from R to 'r_1' is:

$$\mathbf{T}_{tot} = e^{-2\int_R^{r_1} k_2(r)dr}$$

Or $\ln(T_{tot}) = -2\int_R^{r_1} k_2(r)dr$ ……………………………………(2.12)

For r> r_1, E_α is greater than V(r). So if an α particle can get past r_1 it will permanently escape from the nucleus. Since at r =r_1, V(r) = E_α, hence

$E_\alpha = \frac{2Z_d e^2}{4\pi\varepsilon_0 r_1}$ ………………………………(2.13)

Or $\frac{2Z_d e^2}{4\pi\varepsilon_0 r} = \frac{2Z_d e^2}{4\pi\varepsilon_0 r_1} \cdot \frac{r_1}{r} = \frac{E_\alpha}{r}$ ……………………………………(2.14)

Substituting in equation (2.11)

$$K_2(r) = \sqrt{\frac{2M_\alpha E_\alpha}{\hbar^2}} \cdot \sqrt{\frac{r_1}{r} - 1}$$

So $\ln(T_{tot}) = -2\sqrt{\frac{2M_\alpha E_\alpha}{\hbar^2}} \int_R^{r_1} \sqrt{\frac{r_1}{r} - 1}dr$ ……………………………………(2.15)

Substituting r = $r_1 Cos^2\theta$

$$\int_R^{r_1} \sqrt{\frac{r_1}{r} - 1}dr = -r_1\int_R^{r}(1 - Cos2\theta)d\theta$$

$$= -r_1\{\theta - Sin2\theta\}_R^{r_1}$$

$$= -r_1\{\theta - 2Sin\theta Cos\theta\}_R^{r_1}$$

$$= -r_1\left\{Cos^{-1}\sqrt{\left(\frac{R}{r_1}\right)} - 2\sqrt{\frac{R}{r_1}\left(1 - \frac{R}{r_1}\right)}\right\}$$

So $\ln(T_{tot}) = 2r_1\sqrt{\frac{2M_\alpha E_\alpha}{\hbar^2}}\left\{Cos^{-1}\sqrt{\left(\frac{R}{r_1}\right)} - 2\sqrt{\frac{R}{r_1}\left(1 - \frac{R}{r_1}\right)}\right\}$ ………………………………(2.16)

Assuming $r_1 >> R$: $Cos^{-1}\sqrt{\left(\frac{R}{r_1}\right)} = \frac{\pi}{2} - \sqrt{\left(\frac{R}{r_1}\right)}$ and $\sqrt{\left(1-\frac{R}{r_1}\right)} \approx 1$

Hence $\ln(T_{tot}) = 2r_1\sqrt{\frac{2M_\alpha E_\alpha}{\hbar^2}}\left\{\frac{\pi}{2} - 2\sqrt{\left(\frac{R}{r_1}\right)}\right\}$ ……………………..(2.17) Substituting the value of r_1 in terms of E_α from equation (2.13)

$$\ln(T_{tot}) = \frac{4e}{\hbar}\sqrt{\frac{M_\alpha Z_d R}{\pi\varepsilon_0}} - \frac{e^2 Z_d}{\hbar\varepsilon_0}\sqrt{\frac{M_\alpha}{2E_\alpha}} \quad \text{……………………………………..(1.18)}$$

$\frac{4e}{\hbar}\sqrt{\frac{M_\alpha}{\pi\varepsilon_0}} = 2.97$ and $\frac{e^2}{\hbar\varepsilon_0}\sqrt{\frac{M_\alpha}{2}} = 3.95$ as all the quantities involved are universal constants.

$$\therefore \quad \ln(T_{tot}) = 2.97\sqrt{Z_d R} - 3.95 Z_d E_\alpha^{-\frac{1}{2}} \quad \text{……………………………(2.19)}$$

Where R is in Fermi(10^{-15} m) and E_α is in MeV.

From equation (2.7) and (2.19)

$$\ln(\lambda) = \ln\left(\frac{v}{2R}\right) + \ln T_{tot}$$

$$= \ln\left(\frac{v}{2R}\right) + 2.97\sqrt{Z_d R} - 3.95 Z_d E_\alpha^{-\frac{1}{2}} \quad \text{…………………………………….(2.20)}$$

Verification of results: Fig. (2.4) shows a graph of $\ln(\lambda)$ vs. $Z_d E_\alpha^{-\frac{1}{2}}$. The slope of the graph is expected to be 3.95 (equation. 2.20) The intercept on the y axis gives the value of $\ln\left(\frac{v}{2R}\right) + 2.97\sqrt{Z_d R}$. The value of nuclear radius 'R' obtained from the intercept for a given element is of the same order as obtainer experiments like scattering. The expression for $\ln(\lambda)$ shows that $\ln(\lambda)$ and hence λ decreases with increase of E_α. So for high energetic alpha particles the decay constant λ is low and hence the parent nucleus is short lived. For low energetic alpha particles the decay constant λ is high and hence the parent nucleus is long lived.

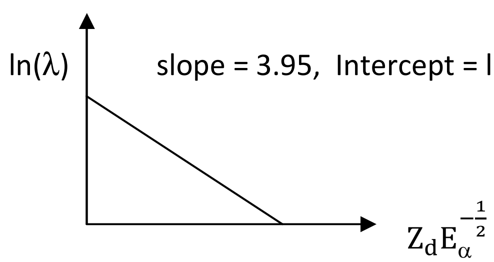


Fig. 2.4. Graph of $\ln(\lambda)$ vs. $Z_d E_\alpha^{-\frac{1}{2}}$

BETA DECAY

A **beta particle**, also called **beta** ray or **beta radiation** (symbol **β**), is a high-energy, high-speed electron or positron (a subatomic particle with the same mass as an electron and a numerically equal but positive charge) emitted by the radioactive decay of an atomic nucleus during the process of **beta** decay. There are two forms of **beta** decay, **β**$^-$ decay and **β**$^+$ decay, which produce electrons and positrons respectively.

In nuclear physics, **beta decay** (**β-decay**) is a type of radioactive **decay** in which a **beta** particle (fast energetic electron or positron) is emitted from an atomic nucleus, transforming the original nuclide to an isobar (elements with same mass number but different atomic number). The mass number of the daughter remains same as that of the parent but the atomic number changes by unity.

Beta decay occurs when, in a nucleus with too many protons or too many neutrons, one of the protons or neutrons is transformed into the other.

In beta minus decay, a neutron decays into a proton and electron:

$$_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0$$

In beta plus decay, a proton decays into a neutron and positron:

$$_1p^1 \rightarrow n^1 + {}_1e^0$$

A beta minus decay is represented by

$$_ZX^A = {}_{Z+1}Y^A + {}_{-1}e^0 \quad \text{.....................................(2.21a)}$$

A beta plus decay is represented by

$$_ZX^A = {}_{Z-1}Y^A + {}_1e^0 \quad \text{.....................................(2.21a)}$$

Examples of β decay reaction

1. β decay of carbon-14 (electron emission)

$$_6C^{14} \rightarrow {}_7N^{14} + {}_{-1}e^0$$

2. β decay of carbon-10 (positron emission)

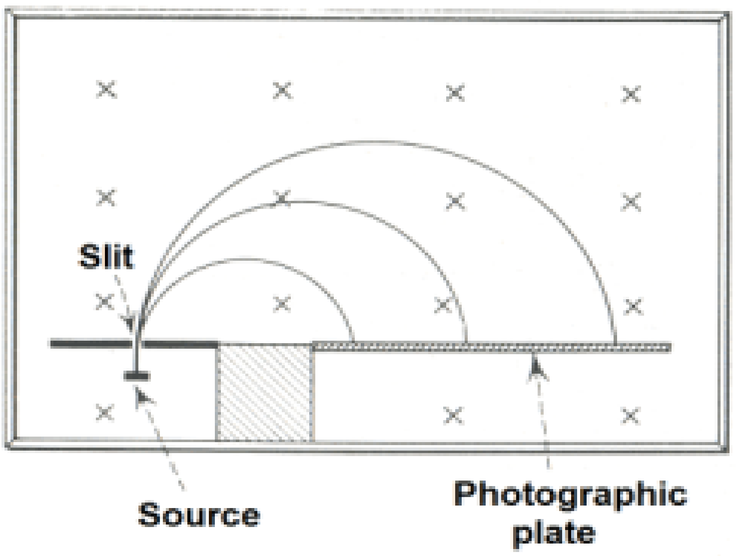
$$_6C^{10} \rightarrow {}_5B^{10} + {}_{-1}e^0$$

β ray energy spectrum

The energy spectrum of β particles has been studied with the help of a magnetic spectrometer shown in the figure below. The radioactive source of beta emitter is placed in a cavity in a block of lead. The whole set up is enveloped in a highly evacuated box. The beta particles emitted from the source are deflected into a circular path of radius 'R' by a magnetic field perpendicular to the vertical plane of the box in accordance with the equation:

$$Bev = \frac{mv^2}{R} \quad \text{...(2.22)}$$

Where 'B' is the magnetic field strength, 'e' is the electron or positron charge, 'm' is the mass of electron and 'v' is its velocity.



Beta-ray spectrometer

Fig. 5

Thus particles of different energies (different velocities) follow different circular paths and are recorded by a photographic plate. From the photographic impression, the relative number of beta particles and their energies can be determined. The figure below shows a graph of intensity of β emission vs. Kinetic energy of β particles

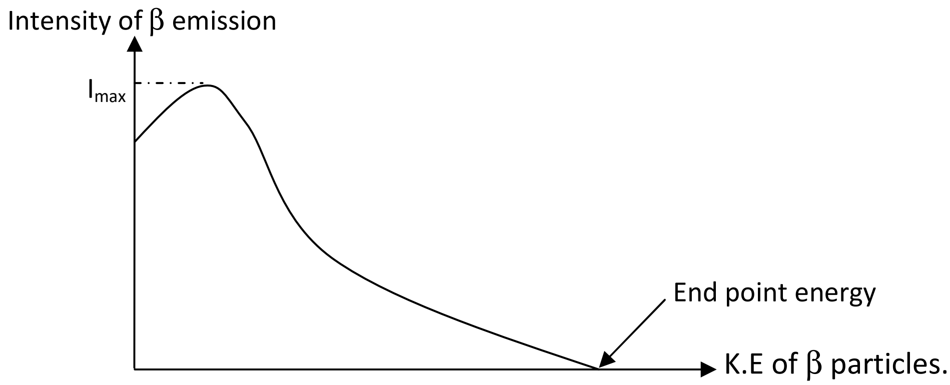


Fig. 2.6. Beta particle spectrum.

A typical β ray spectrum is shown in Fig.(2.7). The graph exhibits that the β particles have a maximum intensity $I_{max.}$ Below I_{max} there is a continuous spectrum with average Intensity usually less than half the maximum. Every continuous β spectrum exhibits a definite maximum, the height and position of which depends on the nucleus emitting β particles.

End point energy – The energy range of β spectrum ranges from zero to a well defined limit called **end point energy,** where the intensity of β emission is zero. The value of this end point energy is the characteristic of the β emitter. This end point energy is the maximum energy with which a β particle is emitted from the source, varying over a wide range throughout the list of beta unstable nuclei being 18keV for tritium to 13 Mev for isotope B^{12}.

Energy equation of β particle

1. When a nucleus emits a β particle a neutron in the parent nucleus changes to a proton thereby increasing the atomic number by unity, keeping the mass number same. The β decay equation is given by:

$$_{Z}X^{A} = {}_{Z+1}Y^{A} + {}_{-1}e^{0}$$

The corresponding energy conservation equation demands that:

$$\mathbf{M_Pc^2 - Zm_ec^2} = M_Dc^2 - (Z+1)m_ec^2 + m_ec^2 + Q \quad \text{...(2.23)}$$

Where M_P = mass of parent atom

M_Pc^2 = corresponding rest mass energy of parent atom

$(M_P - M_D)c^2$ = corresponding rest mass energy of parent nucleus

M_D = rest mass of daughter atom

$M_D c^2$ = corresponding rest mass energy of daughter atom

$M_D c^2 - (Z+1)m_e c^2$ = corresponding rest mass energy of daughter nucleus

m_e = rest mass of β particle = mass of electron

$m_e c^2$ = corresponding rest mass energy of electron

Z = atomic number

Q = kinetic energy of β particle

From equation (2.23)

$(M_P - M_D)c^2 = Q$ ……………………………………….(2.24)

Invalidity of laws of energy and momentum conservation in β decay

Energy conservation: Equation (2.24) shows that for a given source the values of M_P and M_D are fixed, which thereby makes the value of Q constant. This indicates that all β particles from a given radioactive substance must be emitted with the same kinetic energy $(M_P - M_D)c^2$. But actual observations show that only a few β particles are emitted with this energy, while a majority of them are emitted with a smaller energy. Thus the result obtained from **energy conservation relation fails to explain the experimental results.**

Linear momentum conservation: If a parent atom at rest and decays into just two particles, the daughter and β particle, then according to the principle of momentum conservation, the sum of linear momentum of the two decayed particles must be zero. This demands that the β particles and the daughter nucleus must move in a straight line in opposite directions. But experimental observations of individual β particles have shown that their directions of motion are not opposite. **Thus there is an apparent violation of the principle of conservation of linear momentum.**

Angular momentum conservation: The nuclei have angular momentum due to spin of nucleons. According to shell model of nuclear structure, the nuclei with even mass number have spin angular momentum equal to zero or some integer multiple of ħ, while those with odd mass number have a spin angular momentum equal to half odd integer multiple of ħ. In β emission, the mass number, i.e. the number of nucleons in the nucleus remains unchanged. So the angular momentum of parent nucleus and daughter nucleus are same. But due to the spin of emitted β particle the product side has an extra angular momentum of $\frac{\hbar}{2}$ as compared to the parent side. **Thus there is an apparent violation of the principle of conservation of angular momentum.**

Neutrino hypothesis

The problem of apparent violation of the conservation principles was eliminated by Pauli in 1933 by assuming the existence of another additional particle which he named **neutrino** and its antiparticle called **antineutrino. The neutrino (ν) is defined as a particle with is spin vector parallel to its momentum vector and the antineutrino ($\overline{\nu}$) as a particle with is spin vector parallel to its momentum vector**. Both of them are massless and chargeless but has energy E and a spin angular momentum $\frac{\hbar}{2}$. Due to zero rest mass then both of these particles will travel with 'c', the speed of light. Consequently its linear momentum is:

$$p = \frac{E}{c} \quad \text{.................................}(2.25)$$

Neutrino theory of β decay

According to the neutrino theory, the β decay of radioactive nuclei involves the following processes.

$$\left.\begin{array}{ll} \text{For positron emission:} & {}_1p^1 \rightarrow {}_0n^1 + {}_1e^0 + \nu \\ \text{For electron emission} & {}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \overline{\nu} \end{array}\right\} \quad \text{...............................}(2.26)$$

Positron emission involves the spontaneous conversion of a proton to a neutron with the emission of both positron and neutrino. Electron emission involves the spontaneous conversion of a neutron to a proton with the emission of both electron and antineutrino.

So now a beta minus decay is represented by

$${}_ZX^A = {}_{Z+1}Y^A + {}_{-1}e^0 + \overline{\nu} \quad \text{....................................}(2.27)$$

And a beta minus decay is represented by

$${}_ZX^A = {}_{Z+1}Y^A + {}_{-1}e^0 + \nu \quad \text{....................................}(2.28)$$

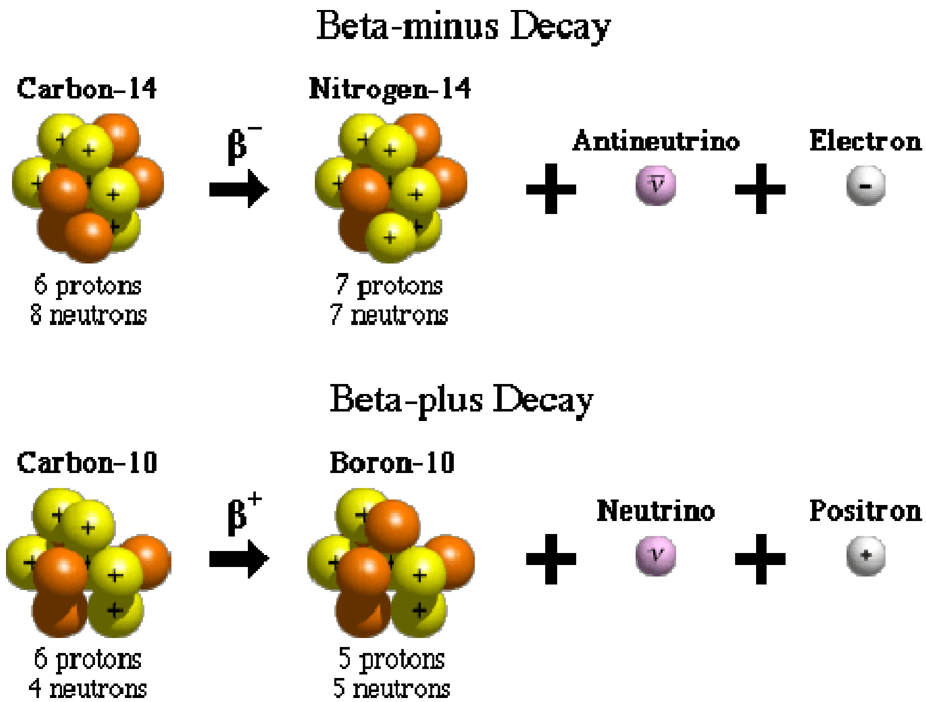
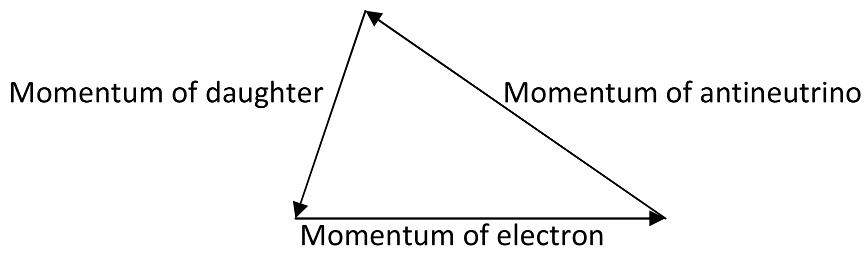


Fig. 2.7. Equation of β emission

Explanation of discrepancies in the conservation laws in β emission

Energy conservation: With the introduction of neutrino it can be assumed that the total disintegration energy Q in equation (2.23) is now carried jointly by the electron and the antineutrino. So the kinetic energy carried by the electron decreases with increase of neutrino energy, being maximum at $(M_P - M_D)c^2$ when the neutrino energy is zero. In all other cases the electron will carry an energy less than the maximum. This maximum energy is **the zero point energy**. **So though the value of Q remains constant, but the energy of the electron can vary over a continuous range and there is no question of violation of conservation of energy.** The same argument is valid for positron emission also.

Linear momentum conservation: In a β decay process from a parent radioactive nucleus, there will be three particles on the product side instead of two. According to principle of conservation of linear momentum the vector sum of the linear momentum of these three particles must be zero. This is possible if the linear momentum vectors of the three particles form three sides of a triangle taken in order.



Hence there is no requirement of the daughter nucleus and the β particle to travel in exactly opposite directions along a same straight line after emission. Instead there can be a variety of ways in which three separate momentum vectors can be arranged to give zero resultant. Hence **there is no question of violation of conservation of linear momentum.** The same argument is valid for positron emission also

Angular momentum conservation: The spin angular momentum of antineutrino is $\frac{\hbar}{2}$. Therefore when an electron and an antineutrino is ejected simultaneously, then spin of the electron is complemented by the spin of the antineutrino. Consequently the resultant angular momentum of all the particles left after decay will be that due to the daughter nucleus which is same as that of the parent. **Thus the total angular momentum is conserved.**