MATTER WAVES

The particle nature of light was established by the introduction of the concept of photons – the quantized light. The radiation quantum hypothesis was used by Einstein to explain photoelectric effect. Later this concept was also used to explain Compton effect. But to explain phenomena like interference, diffraction and polarization the wave nature of light has to be considered. So it can be said that light has a dual nature – the wave-particle duality.

de Broglie extended the dual nature of radiation to material particles on the basis of the following reasonings:

1. Nature is strikingly symmetric in many respects.

2 The observable universe is composed entirely of radiation and matter

3. If light possesses duality perhaps the sub –atomic particles should also possess duality.

In 1924, Louis de Broglie proposed a new speculative hypothesis that electrons and other particles of matter can behave like waves. Today, this idea is known as de Broglie's hypothesis of matter waves. According to de Broglie's hypothesis, massless photons as well as massive particles must satisfy one common set of relations that connect the energy E with the frequency f, and the linear momentum p with the wavelength λ. Any particle that has energy 'E' and momentum 'p' is a de Broglie wave of frequency ν and wavelength λ. If E be the energy and p be the momentum then

$$E = h\nu$$

And $$p = \frac{h}{\lambda}$$

Hence $$\lambda = \frac{h}{p} \quad \text{..............................(4.1)}$$

So a material particle of mass 'm' while in motion with a velocity 'v' will have associated with it, a matter wave of wavelength given by :

$$\boxed{\lambda = \frac{h}{p} = \frac{h}{mv}} \quad \text{...(4.2)}$$

This wavelength is referred to as the de Broglie wavelength of a particle in motion. The relation (4.2) is called the de Broglie relation.

BRAGG'S LAW OF REFLECTION OF X RAYS

(Not directly included in the syllabus but required for understanding Davisson-Germer experimental results)

When a crystal is illuminated by X rays , the energy of X rays drives the atomic electrons to vibration with a frequency equal to that of the incident radiation. Since an accelerating charge emits radiation, hence the vibrating electrons inside the crystal act as sources of secondary waves which proceed in all directions through the crystal. This phenomenon is also termed as **scattering of X rays by atomic electrons.** If the wavelength of incident radiation is large compared to the atomic dimension and inter-atomic spacing, then the radiation passing through the crystal is not perturbed by the atoms. So, all the secondary radiations are in phase with each other. But actually the wavelength of X rays is comparable with the atomic dimension and inter-atomic spacing. Hence the radiation emitted by the electrons, are out of phase with each other thereby producing constructive and destructive interference between them, generating maximas and minimas in certain directions. These directions correspond to different orders of X ray diffraction similar to grating. The atoms serve as opaque spaces and the inter-atomic spacing serve as slits of the reflection grating. The condition for a crystal to diffract X rays can be obtained from **Bragg's treatment**

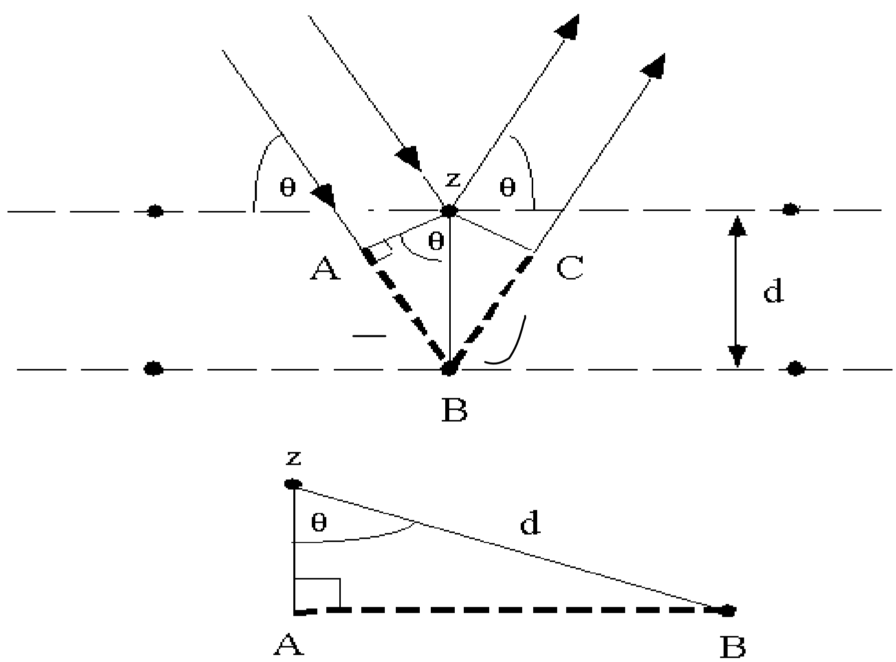

Fig: 4.1. Bragg's reflection of X rays from atomic planes

In 1912 W.H Bragg and W.L Bragg put forward a model which generates a condition for diffraction of radiation from atomic planes. They pointed out that a crystal may be divided into a set of parallel planes which are capable of diffracting X rays. The reflected X rays produce constructive interference for certain direction of incidence, the corresponding glancing angle being called the **Bragg angle.** Based on this approach Bragg derived a simple mathematical relation for **Bragg condition of reflection**. This condition is also known as the Bragg's law.

To obtain Bragg's law of reflection a set of crystal planes are considered with interplaner spacing 'd'. Z and B are two atoms in adjacent planes which reflect incident X rays as shown in fig. (4.1). So ZB = d = interatomic spacing. The X rays are incident at a glancing angle θ'. Consequently the glancing angle for the reflected rays is also 'θ' . Referring to fig. (4.1):

Path difference between the two reflected rays:

$$\Delta x = ZA + ZC = ZB.\mathrm{Sin}(\theta) + ZB.\mathrm{Sin}(\theta) = 2\, ZB.\mathrm{Sin}(\theta) = 2d\mathrm{Sin}(\theta) \quad \text{.........(4.3)}$$

For a maxima to occur $\Delta x = n\lambda$, where n is any integer. So we have

$$\boxed{2d\mathrm{Sin}(\theta) = n\lambda} \quad \text{...............................(4.4)}$$

Equation (4.4) is called the Bragg equation. This equation immediately shows that for given 'd' and 'λ' and 'n' there is only one definite value of glancing angle 'θ' which satisfies the Bragg equation. The equation also shows that if: $\lambda > 2d$the value of $\sin(\theta)$ will be > 1, which is not possible. Hence only those radiations whose wavelength is comparable or less than the interplaner spacing of crystals can be used for studying a crystal structure via difrfraction. X rays satisfiy this criterion.

<u>**DAVISSON – GERMER EXPERIMENT**</u>

de Broglie's idea of matter waves was confirmrd in 1927 by Davisson and Germer through diffraction of a beam of slow electrons by a crystal. The associated de Broglie wavelength for such electrons is of the order of inter atomic spacingsof most of the crystalline solids.Atoms of a crystal can serve as a three dimensional array of diffracting centres for matter waves.

Experimental set up

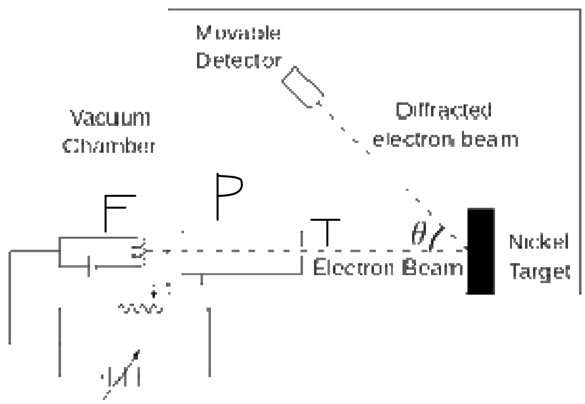

Fig. 4.2: Experimental set up of Davisson Germer experiment

The experimental setup for the Davisson and Germer experiment is enclosed within a vacuum chamber. Thus the deflection and scattering of electrons by the medium are prevented. The main parts of the experimental setup are as follows:

1. Electron gun F: An electron gun is a thermally heated Tungsten filament that emits electrons via thermionic emission i.e. it emits electrons when heated to a particular temperature.

2. Electrostatic particle accelerator P: Two opposite charged plates (positive and negative plate) are used to accelerate the electrons at a known potential.

3. Collimator T : The accelerator is enclosed within a cylinder that has a narrow passage for the electrons along its axis. Its function is to render a narrow and straight (collimated) beam of electrons ready for acceleration.

4. Target: The target is a Nickel crystal. The electron beam is fired normally on the Nickel crystal. The crystal is placed such that it can be rotated about a fixed axis.

5. Detector: A detector is used to capture the scattered electrons from the Ni crystal. The detector can be moved in a semicircular arc to any angle θ, relative to the incident beam as shown in the diagram above.

Davisson and Germer measured the intensity of scattered electrons at different values of θ. The variation in intensity plotted as a function of θ generates a graph called the polar graph or polar plot

Observations

The detector used here can only detect the presence of an electron in the form of a particle. As a result, the detector receives the electrons in the form of an electronic current. The intensity (strength) of this electronic current received by the detector and the scattering angle is studied. We call this current as the electron intensity.

The intensity (I) of the scattered electrons is not continuous. It shows a maximum and a minimum value corresponding to the maxima and the minima of a diffraction pattern produced by X-rays. It is studied from various angles of scattering and potential difference. For a particular voltage (54V, say) the maximum scattering happens at a fixed angle only (50^0) as shown below:

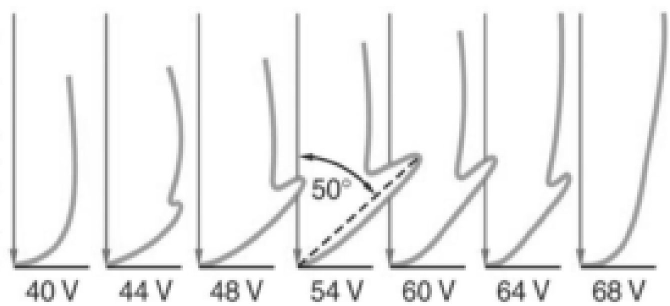

Fig. 4.3. Plots between the intensity (I) of scattering (X-axis) and the angle of scattering θ for given values of Potential difference.

Results of the Davisson and Germer Experiment

If 'V' be the voltage between the plates of the particle accelerator, 'e' be the charge on electron, and 'm' be its massthen the energy of the electron is only kinetic:

$$E = \frac{p^2}{2m} = eV, \text{ which gives}$$

$$p = \sqrt{2mE} = \sqrt{2meV} \quad \text{.................................}(4.5)$$

from the de Broglie equation we have:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} \quad \text{............................}(4.6)$$

For V = 54 volt we have: $\lambda = 0.167$ nm ………………(4.7)

This is the theoretical value of 'λ' obtained from de Broglie equation.

This value of 'λ' is obtained from Davisson Germer experiment.

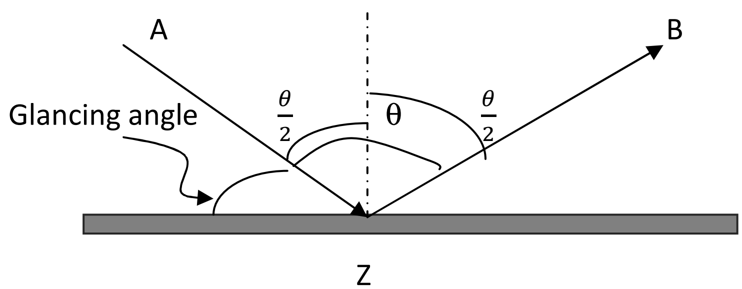

Fig. 4.3. Scattering of electron by nickel crystal

AZ is the incident beam of electrons, which are scattered along ZB due to reflection. In fig.(4.3) the scattering angle is 'θ' So the angle of incidence of electrons on the Nickel target is = $\frac{\theta}{2}$, the scattering angle being the sum of angle of incidence and angle of reflection and hence equal to twice the angle of incidence.

Hence glancing angle = 90^0 – angle of incidence = $\left(90^0 - \frac{\theta}{2}\right)$

Hence applying Bragg equation:

$$2d\text{Sin}\left(90^0 - \frac{\theta}{2}\right) = n\lambda \quad \text{………………………………….(4.8)}$$

Now the value of 'd' for nickel crystal as obtained from X-ray scattering is 0.092 nm. For V = 54 V, the angle of scattering θ = 50^0, we have from equation (4.8)

$$2\times0.092\times\text{Sin}\left(90^0 - \frac{\theta}{2}\right) = n\lambda$$

For n = 1, we have:

$\lambda = 0.165$ nm ……………………………… (4.9)

This value of λ' is obtained from Experimental results.

Therefore the experimental results are in a close agreement with the theoretical values got from the de Broglie equation. The equations (4.7) and (4.9) verify the de Broglie equation thereby establishing the wave nature of particles.

From the Davisson and Germer experiment, we get a value for the scattering angle θ and a corresponding value of the potential difference V, at which the scattering of electrons is maximum. Thus these two values from the data collected by Davisson and Germer, when used in equation (1) and (2) give the same values for λ. Therefore, this establishes the de Broglie's wave-particle duality and verifies his equation as shown below: