INTERFERENCE IN THIN FILMS

I: INTERFERENCE BY A PARALLEL SIDED THIN FILM

When a thin film of some transparent material is exposed to light, circular patterns of brilliant colours can be seen for e.g. colours seen in a soap bubble, or formation of coloured rings on a wet road on which car oil has been spilled. This is due to interference of light reflected from upper and lower surface of the film via the method of division of amplitude.

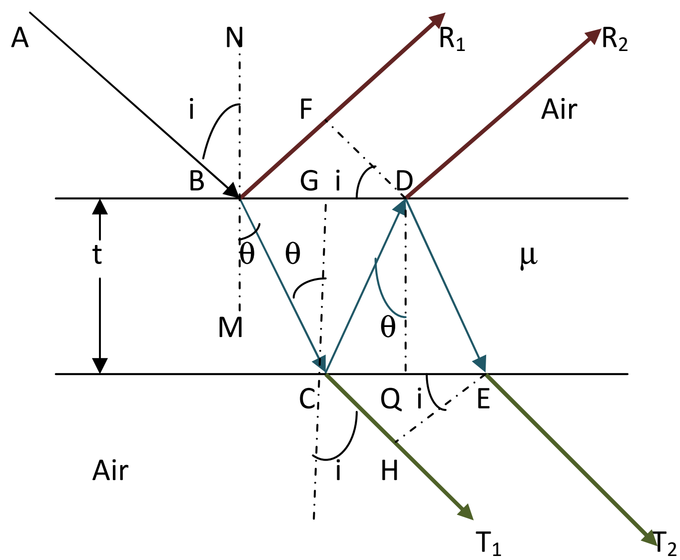


Fig. 5.1. Interference of light by parallel sided thin film.

Let a parallel sided thin film of refractive index μ and thickness, ‘t’ be considered within an air medium. It is assumed surfaces of the film have a high reflection coefficient

AB $\rightarrow$ A beam of light of amplitude ‘a’ incident on the upper surface of the film.

$BR_1 \rightarrow$ Light reflected from the upper surface of the film.

BC $\rightarrow$ Light transmitted into the film through the upper surface.

CD $\rightarrow$ Light reflected from the lower surface of the film.

$CT_1 \rightarrow$ Light transmitted outside the film through the lower surface

$DR_2 \rightarrow$ Light transmitted outside the film through the upper surface

DE $\rightarrow$ Light re-reflected into the film at the point D

$ET_2 \rightarrow$ Corresponding transmitted light.

The reflection coefficient of the film surfaces being high it can be assumed that the amplitude becomes insignificant after two reflections and hence other subsequent reflections inside the film can be ignored.

FD and EH are perpendiculars drawn from D and E on BR_1 and CT_1 respectively

CG is the perpendicular drawn from C on BD.

$\angle ABN = i$ = Angle of incidence = $\angle FDB = \angle CEH$ from the geometry of the fig. (5.1)

$\angle MBC = \theta$ = Angle of refraction = $\angle GCB = \angle CDQ$ from the geometry of the fig. (5.1)

The reflected beams BR_1 and DR_2 are parts of the same wave obtained by division of amplitude. So they are coherent and will interfere. Also the transmitted beams CT_1 and ET_2 are parts of the same wave obtained by division of amplitude. So they are coherent and will interfere. Hence both for reflected beams and transmitted beams interference fringes will be formed. Since the reflected beams are parallel, the interfering beams superpose at infinity. Hence the fringes are formed at infinity. The same thing holds for transmitted beam. So to observe the fringes a convex lens is required to focus the parallel beams at its focal plane.

Path difference and conditions of maxima and minima for reflected beams

The beam DR_2 travels through the thin film twice before emerging from it. So it travels an extra optical path: $\mu(BC + CD)$ as compared to BR_1. On the other hand BR_1 travels an extra path BF in air as compared to DR_2. So the net optical path difference between the beams BR_1 and DR_2 is:

$$\Delta x = \mu(BC + CD) - BF$$

Now, from the geometry of the Fig. (5.1) BC = CD, which gives

$$\Delta x = 2\mu BC - BF$$

G is the midpoint of BD and CG is perpendicular to BD. CG = t. This gives

$$\Delta x = 2\mu.CG.Sec\theta - BD.Sin(i)$$

From Snell's law of refraction $Sin(i) = \mu Sin\theta$

$\therefore \quad \Delta x = 2\mu.CG.Sec\theta - \mu.BD.\ Sin\theta$

Or $\quad \Delta x = 2\mu.CG.Sec\theta - 2\mu BG.Sin\theta \qquad$ as BD = 2BG

Or $\Delta x = 2\mu.CG.Sec\theta - 2\mu CG.\tan\theta.Sin\theta$

Or $\Delta x = 2\mu.t.Sec\theta - 2\mu t.Sin^2\theta.Sec\theta$

Or $\Delta x = 2\mu.t.Sec\theta.(1 - Sin^2\theta)$

Or $\Delta x = 2\mu.t.Sec\theta.Cos^2\theta$

Or $\boxed{\Delta x = 2\mu t.Cos\theta}$ ……………………………………(5.1)

Equation (5.1) gives the path difference between the reflected interfering beams BR_1 and DR_2

BR_1 is reflected at the surface of a denser medium. So it suffers a phase change of π, while the beam DR_2 comes via reflection at C i.e. at the surface of a rarer medium and hence suffers no phase change. Hence between the beams BR_1 and DR_2 an extra phase of π occurs other than that due to the path difference. So net phase difference between the two reflected beams is:

$\Delta\phi = \frac{2\pi}{\lambda} . 2\mu t Cos\theta - \pi$ (Note: $+\pi$ can also be used)

Condition for maxima

$\Delta\phi = 2n\pi$

Or $\frac{2\pi}{\lambda}.2\mu t Cos\theta - \pi = 2n\pi$ where n = 0, 1, 2,…….

Or $\boxed{2\mu t Cos\theta = (2n + 1)\frac{\lambda}{2}}$ ………………………………………(5.2)

Condition for minima

$\Delta\phi = (2n - 1)\pi$

Or $\frac{2\pi}{\lambda}.2\mu t Cos\theta - \pi = (2n - 1)\pi$ where n = 1, 2, 3…..

Or $\boxed{2\mu t Cos\theta = n\lambda}$ …………………………………………..(5.3)

If instead of air the film is placed in a medium which is optically denser than the material of the film: When light travelling through a denser medium is reflected at the surface of a rarer medium, the reflected beam BR_1 will not undergo any phase change. The beam DR_2 comes via reflection at C i.e. at the surface of a rarer medium and hence suffers a phase change of π. So in this case also between the beams BR_1 and DR_2 an extra phase of π occurs other than that due to the path difference. Hence the conditions of maxima and minima remain unaltered.

Path difference and conditions of maxima and minima for transmitted beams

The beam ET_2 travels through the thin film twice before emerging from it. So it travels an extra optical path: $\mu(CD + DE)$ as compared to CT_1. On the other hand CT_1 travels an extra path CH in air as compared to ET_2. So the net optical path difference between the beams CT_1 and ET_2 is:

$$\Delta x = \mu(CD + DE) - CH$$

Now, from the geometry of the Fig. (5.1) BC = CD, which gives

$$\Delta x = 2\mu CQ - CH$$

Q is the midpoint of CE and DQis perpendicular to CE. DQ = t. This gives

$$\Delta x = 2\mu.DQ.Sec\theta - CE.Sin(i)$$

From Snell's law of refraction $Sin(i) = \mu Sin\theta$

$\therefore$ $$\Delta x = 2\mu.DQ.Sec\theta - \mu.CE.\ Sin\theta$$

Or $$\Delta x = 2\mu.DQ.Sec\theta - 2\mu CQ.Sin\theta \qquad \text{as } BD = 2BG$$

Or $$\Delta x = 2\mu.DQ.Sec\theta - 2\mu DQ.tan\theta.Sin\theta$$

Or $$\Delta x = 2\mu.t.Sec\theta - 2\mu t.Sin^2\theta.Sec\theta$$

Or $$\Delta x = 2\mu.t.Sec\theta.(1 - Sin^2\theta)$$

Or $$\Delta x = 2\mu.t.Sec\theta.Cos^2\theta$$

Or $$\boxed{\Delta x = 2\mu t.Cos\theta} \quad \text{...(5.4)}$$

Equation (5.1) gives the path difference between the transmitted interfering beams CT_1 and ET_2

CT_1 suffers no reflection in coming from A to T_1. So it undergoes no phase change of π. The beam ET_2 comes via reflection at two points C and D both at the surface of a rarer medium and hence suffers no phase change. Hence between the beams CT_1 and ET_2 the only phase difference is that due to the path difference. So net phase difference between the two transmitted beams is:

$$\Delta\phi = \frac{2\pi}{\lambda}.\ 2\mu t Cos\theta$$

Condition for maxima

$$\Delta\phi = 2n\pi$$

Or $\frac{2\pi}{\lambda}.2\mu t\text{Cos}\theta = 2n\pi$ where n = 0, 1, 2,…….

Or $\boxed{2\mu t\text{Cos}\theta = n\lambda}$ ……………………………………….(5.5)

Condition for minima

$\Delta\phi = (2n - 1)\pi$, where n = 1,2,3 Note: + sign can also be used but for that case n = 0,1,2 ….)

Or $\frac{2\pi}{\lambda}.2\mu t\text{Cos}\theta = (2n - 1)\pi$

Or $\boxed{2\mu t\text{Cos}\theta = (2n - 1)\frac{\lambda}{2}}$ ……………………………………….(5.6)

So it is seen, that the condition for maxima in reflected beams is same as the condition for minima in transmitted beams and vice versa. So reflected and transmitted fringes, are complementary to each other. θ decreases with increase of order. So the order of fringe decreases outwards.

Nature of the fringes

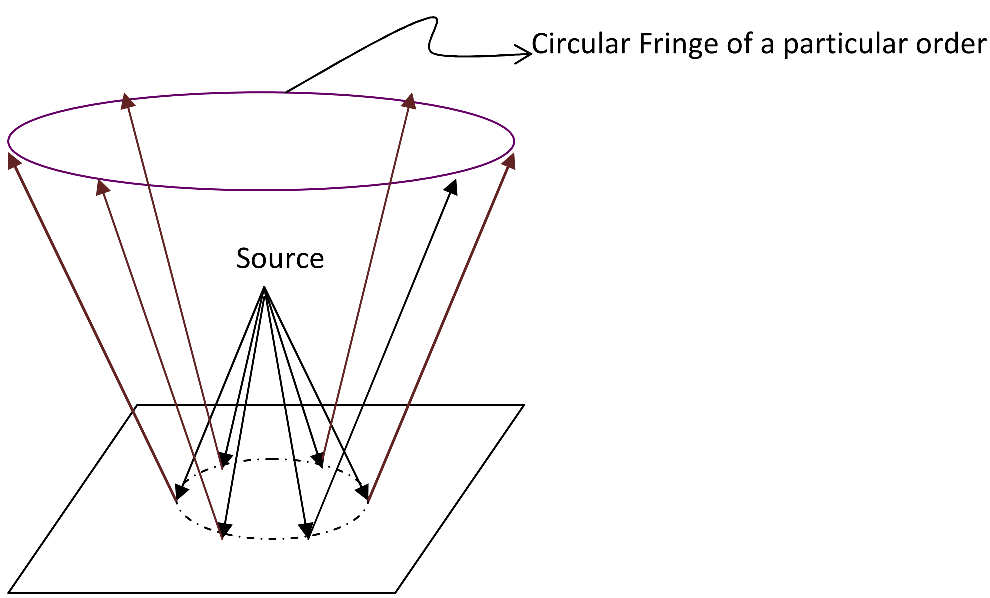


Fig. 5.2. Shape of the fringes in a parallel sided thin film.

In the expression of path difference, for both reflected and transmitted beams, thickness of the film 't' is constant for a parallel sided film. So for a given λ, the fringe order 'n' depends directly on Cosθ, and hence on 'θ' So fringe of a particular order is the locus of all points having the same value of 'θ'. Now, 'θ' is the angle of refraction. So for 'θ' to remain fixed, the angle of incidence should also remain fixed. The locus of all points on the surface of the film, at which angle of incidence of the beams coming from a single source, is constant, is the point of intersection of a cone of semi vertical angle 'i' with the film surface and hence is a circle on the film surface. If all the corresponding reflected beams are collected corresponding to a fixed angle of incidence a circular fringe will be obtained. The fringe pattern for different angles of incidence will be series of concentric bright and dark circles as shown in Fig. (5.3). The same thing holds for transmitted beams also**.** These fringes are called **fringes of equal inclination or Haidinger fringes.**

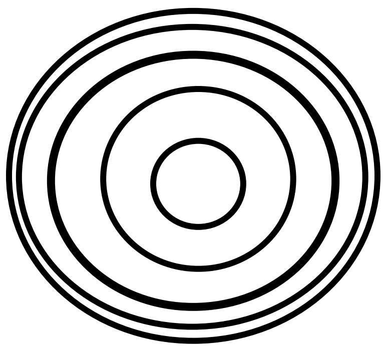


Fig. 5.3. The nature of fringes with fringe width and order decreasing outwards.

For a polychromatic light source

- The refractive index 'μ' and thickness, 't' is constant. So, for a particular order, 'θ' changes with wave length λ'. Hence for each wavelength of a polychromatic light a circular pattern will be obtained for a given order. Hence corresponding to one order, a number of circular fringes of different colours will be obtained as shown in Fig. (5.4).

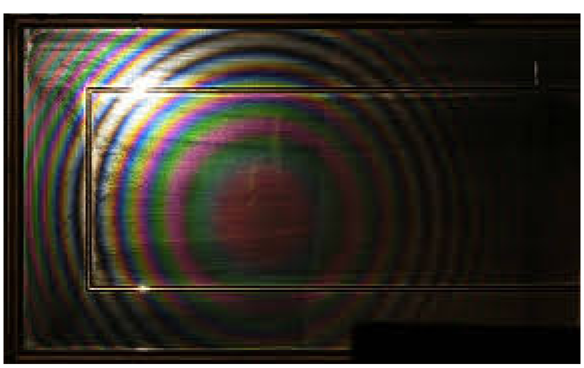


Fig. 5.4. Coloured fringes in thin films

Zero order fringe

For zero order corresponds to zero path difference. The path difference $2\mu t\cos\theta$ will be zero only if $\cos\theta$ is equal to zero, i.e. $\theta = 90^0$

When light coming from a rarer medium is incident on the surface of a denser medium the angle of refraction is always less than the angle of incidence. The maximum possible value of angle of incidence can be 90^0. So the maximum possible angle of refraction is less than 90^0. Hence it is never possible for $\cos\theta$ to be zero thereby making zero order fringe impossible.

When light coming from a denser medium is incident on the surface of a rarer medium the angle of refraction is always greater than the angle of incidence. So for an angle of incidence less than 90^0 (critical angle)the angle of refraction can be equal to 90^0. So it is possible for $\cos\theta$ to be zero thereby making zero order fringe possible. **For reflected fringes the zero order fringe is a dark fringe, while for transmitted fringes the zero order fringe is a bright fringe as is evident from the maxima and minima conditions.**

Angular Fringe width: It is the difference of θ corresponding to two consecutive maximas or two consecutive minimas

The condition of minima for reflected fringe is: $2\mu t\cos\theta = n\lambda$

Differentiating : $-2\mu t\sin\theta.\Delta\theta = \lambda\Delta n$

Considering magnitude only and ignoring the negative sign of differentiation of $\cos\theta$

$$2\mu t\sin\theta.\Delta\theta = \lambda\Delta n$$

For consecutive orders $\Delta n = 1$, which gives

Angular Fringe width = $\boxed{\Delta\theta = \frac{\lambda}{2\mu t\sin\theta}}$ ……………………………..(5.7)

This shows that fringe width decreases with increase of θ i.e. with decrease in order of fringe. So as one moves outwards the fringes become narrower as shown in fig. (3.4). Same result will be obtained for transmitted fringes also.

Nature of central fringe: The central fringe corresponds to the maximum possible path difference: $\Delta x = 2\mu t$. Whether it satisfies the maxima or minima condition will depend on the value of λ. So the central fringe may be bright or dark depending on values of μ, t and λ.

Effect of changing the thickness of the film

From the expression for maxima and minima it is seen that the order of the fringe is directly proportional to 't', the thickness of the film. If the thickness is gradually increased, then condition of maxima or minima of a particular order will be satisfied for a larger value of 'θ'. Consequently the central fringe will shift outwards to make room for the next higher order fringe. Hence the entire pattern seems to move outwards from inside.

If the thickness is gradually decreased, then condition of maxima or minima of a particular order will be satisfied for a lower value of 'θ'. Consequently the central fringe will shift inwards to make room for the next lower order fringe. Hence the entire pattern seems to move inwards from outside.

For an Ultrathin film

For an ultrathin film, the thickness of the film is negligible compared to the wave length of light. So as 't' $\rightarrow$ 0, the path difference $\Delta x = 2\mu t \cos\theta$ can practically taken to be zero as compared to wave length for all values of θ. So $\Delta x = 0$ corresponds to a zero order minima for reflected fringes for all 'θ. Hence instead of a fringe pattern, a uniform darkness will be obtained throughout the field of view. This explains the darkening of a thin soap bubble just before it explodes.

The transmitted fringes being complementary to the reflected fringes, it will exhibit an uniform brightness throughout the field of view, indicating that the energy absent in reflected light is fully present in transmitted light.

Difference between Reflected and transmitted fringes for a parallel sided thin film

Serial no.	Reflected fringes	Transmitted fringes
1.	Condition for maxima is $2\mu t \cos\theta = (2n + 1)\frac{\lambda}{2}$	Condition for maxima is : $2\mu t \cos\theta = n\lambda$
2	Condition for minima is : $2\mu t \cos\theta = n\lambda$	Condition for mniima is $2\mu t \cos\theta = (2n + 1)\frac{\lambda}{2}$
3	The zero order fringe if present is a dark fringe.	The zero order fringe if present is a bright fringe.

II. INTERFERENCE BY A WEDGE SHAPED FILM

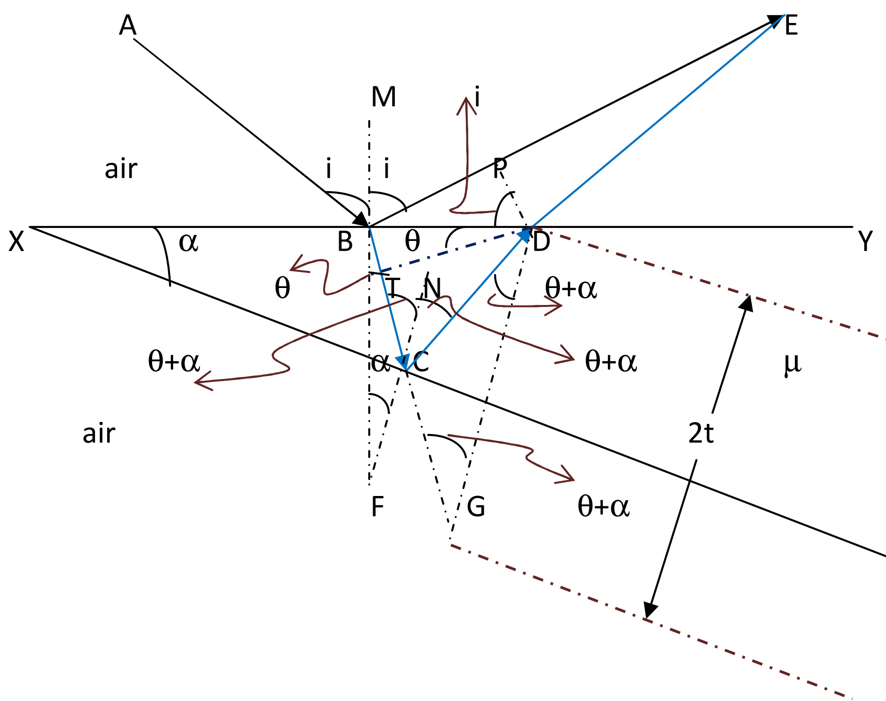


Fig. 5.5 . Interference by a wedge shaped thin film.

When a wedge shaped thin film of some transparent material is exposed to light, straight line patterns of brilliant colours can be seen for This is due to interference of light reflected from upper and lower surface of the film via the method of division of amplitude.

Let wedge shaped thin film YXZ **of refractive index μ and angle of wedge 'α'** be considered within an air medium. It is assumed surfaces of the film have a high reflection coefficient

AB ⟶ A beam of light of amplitude 'a' incident on the upper surface of the film.

BE ⟶ Light reflected from the upper surface of the film.

BC ⟶ Light transmitted into the film through the upper surface.

CD ⟶ Light reflected from the lower surface of the film.

DE ⟶ Light transmitted outside the film through the upper surface

MF ⟶ Normal to XY

NF, DG $\rightarrow$ Normal to XZ

$\angle$MFN $\rightarrow$ angles between the normals to the two surfaces of the wedge = α

BC produced meets the normal DG at G

DG $\rightarrow$ Twice the thickness of the film at point D = 2t

t $\rightarrow$ Thickness of the film at point D

i = angle of incidence = $\angle$ABM = $\angle$BME = Angle of reflection

DP $\rightarrow$ perpendicular to BE

DT $\rightarrow$ Perpendicular to BC

$\theta \rightarrow \angle$FBC = Angle of refraction

From the geometry of the Fig. (5.5)

$$\left.\angle PDB = i, \quad \angle BDT = \theta, \quad \angle BCN = \angle NCD = \angle CDG = \angle DGC = \theta+\alpha \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5.8)$$

Hence CD = CG ……………………………….(5.9)

The beam DE travels through the thin film twice before emerging from it. So it travels an extra optical path: μ(BC + CD) as compared to BE. On the other hand BE travels an extra path BF in air as compared to DR_2. So the net optical path difference between the beams BE and DE is:

$$\Delta x = \mu(BC + CD) - BP$$

Or $\quad \Delta x = \mu(BC + CG) - BP \quad$ from (5.9)

Or $\quad \Delta x = \mu.BG - BP$

Or $\quad \Delta x = \mu.(BT + TG) - BD.\mathrm{Sin}(i)$ ……………….from the fig. (5.5)

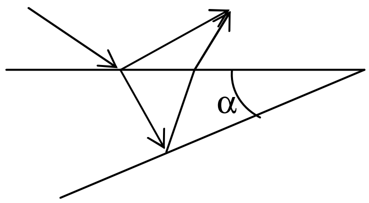
Or $\quad \Delta x = \mu\{BD.\mathrm{Sin}\theta + DG.\mathrm{Cos}(\theta+\alpha)\} - \mu BD.\mathrm{Sin}\theta$

Or $\quad \Delta x = \mu BD.\mathrm{Sin}\theta + \mu\, DG.\mathrm{Cos}(\theta+\alpha) - \mu BD.\mathrm{Sin}\theta$

Or $\quad \Delta x = \mu\, DG.\mathrm{Cos}(\theta+\alpha)$

Or $\quad \boxed{\Delta x = 2\mu t\mathrm{Cos}(\theta+\alpha)}$ ……………………………………………..(5.10)

Equation (5.10) gives the path difference between the reflected interfering beams BE and DE {Please note that in some books the expression of path difference is given by $\Delta x = 2\mu t\cos(\theta-\alpha)$. This expression is obtained if the wedge is reversed laterally as shown in the figure below. Both the expressions are correct}



BE is reflected at the surface of a denser medium. So it suffers a phase change of π, while the beam DE comes via reflection at C i.e. at the surface of a rarer medium and hence suffers no phase change. Hence between the beams BE and DE an extra phase of π occurs other than that due to the path difference. So net phase difference between the two reflected

$$\Delta\phi = \frac{2\pi}{\lambda} \cdot 2\mu t\cos(\theta+\alpha) - \pi \qquad \text{(Note: } +\pi \text{ can also be used)}$$

Condition for maxima

$\Delta\phi = 2n\pi$

Or $\frac{2\pi}{\lambda} \cdot 2\mu t\cos(\theta+\alpha) - \pi = 2n\pi$ where n = 0, 1, 2,…….

Or
$$\boxed{2\mu t\cos(\theta+\alpha) = (2n+1)\frac{\lambda}{2}} \qquad \text{……………………………………(5.11)}$$

Condition for minima

$$\Delta\phi = (2n-1)\pi$$

Or $\frac{2\pi}{\lambda} \cdot 2\mu t\cos(\theta+\alpha) - \pi = (2n-1)\pi$ where n = 1, 2, 3…..

Or
$$\boxed{2\mu t\cos(\theta+\alpha) = n\lambda} \qquad \text{…………………………………………(5.12)}$$

Nature of the fringes

In the expression for path difference it is seen that there are two parameters which may change. The thickness 't' of the film varies from point to point and also the angle of incidence may change causing a change of θ. It is cumbersome to deal with the two changing parameters together. Hence it is advisable to make one of them fixed. Nothing can be done about the thickness for a wedge shaped film. So the only choice is to make the inclination of the incident beam fixed, which will make 'θ' constant for a particular set up. For constant 'θ', the expressions of maxima and minima, shows that the order of the fringe, is directly proportional to the thickness at the concerned point. Hence fringe of a particular order will be the locus of points having constant thickness. This locus is a straight line parallel to the edge of the wedge. So fringes will be alternate bright and dark straight lines parallel to the edge of the wedge as shown in Fig. (5.6). The fringes are **localized fringes** since the interfering occurs at the exit surface of the film. **These fringes are called fringes are called fringes of equal thickness or Fizeau fringes**

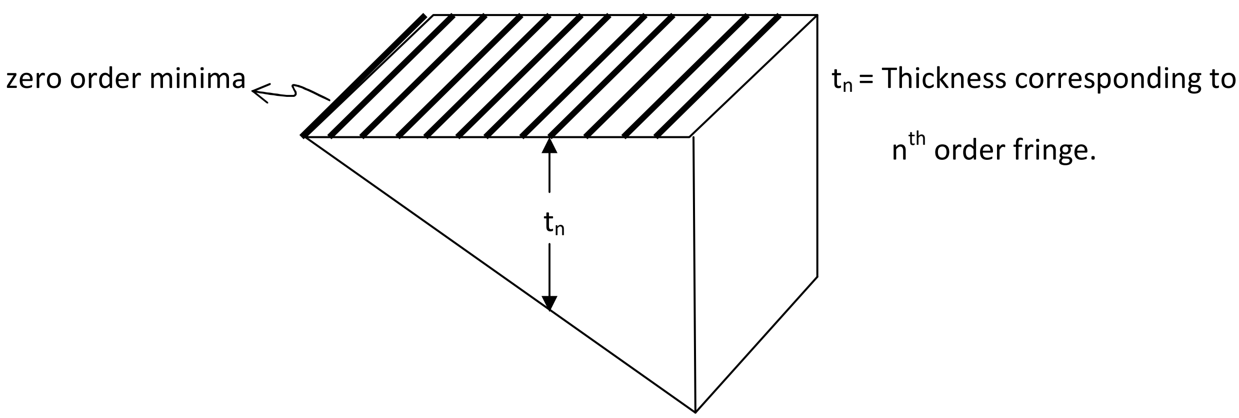


Fig. 5.6. Interference fringes for a wedge shaped thin film

For normal incidence

The most convenient direction of incidence is normal incidence. For normal incidence the angle of incidence 'i' and hence 'θ'is zero. Hence the conditions of maxima and minima reduces to :

$$\left.\begin{aligned} \text{Maxima :} \quad & 2\mu t\text{Cos}\alpha = (2n + 1)\frac{\lambda}{2} \\ \text{Minima :} \quad & 2\mu t\text{Cos}\alpha = n\lambda \end{aligned}\right\} \quad \text{.................................}(5.13)$$

Nevertheless the nature of the fringes remain the same.

Zero order fringe

The zero order fringe will occur at the positions where thickness of the film is zero. This position is the edge of the wedge. Due to the phase change of π due to reflection the zero order will be a dark fringe. So the zero order dark fringe will be formed at the edge of the wedge as shown in Fig. (5.6).

Determination of fringe width

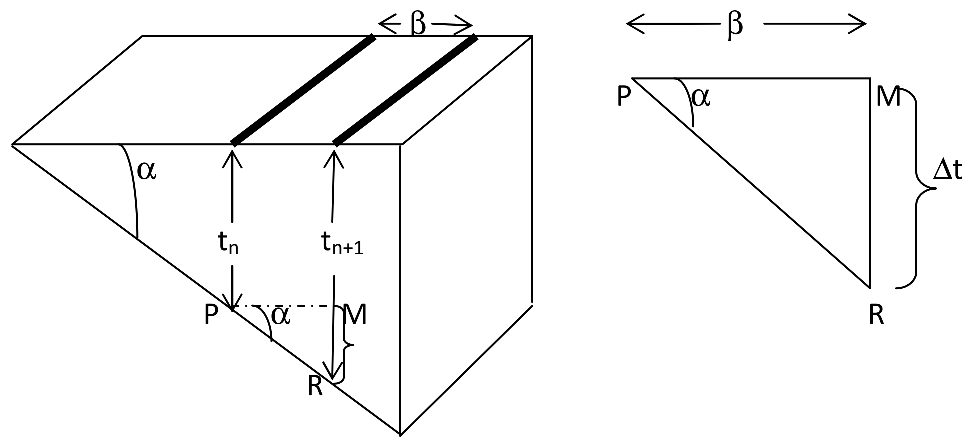


Fig. 5.7. Diagram for calculation of fringr width β

Fringe width is the separation between two consecutive maximas or two consecutive minimas**.** As a fringe of a particular order is the locus of points with constant thickness, hence the separation (β) corresponding to the difference of thickness (Δt) between two consecutive maximas or two consecutive minimas gives the fringe width.

Condition for minima: $2\mu t\mathrm{Cos}(\theta+\alpha) = n\lambda$

Differentiating: $2\mu.\Delta t\mathrm{Cos}(\theta+\alpha) = \Delta n\lambda$

For consecutive orders $\Delta n = 1$, which gives

$$\Delta t = \frac{\lambda}{2\mu\mathrm{Cos}(\theta+\alpha)} \quad \text{...(5.14)}$$

Now from Fig. (5.7) : $\Delta t = \beta.\tan\alpha$ ....(5.15)where β is the fringe width.

Assuming α to be very small for a thin wedge, $\tan\alpha = \alpha$, which gives

$$\Delta t = \beta.\alpha \quad \text{...(5.15)}$$

From equations (5.14) and (5.15) the value of fringe width is

$$\boxed{\beta = \frac{\lambda}{2\mu\alpha\mathrm{Cos}(\theta+\alpha)}} \quad \text{..................................(5.16)}$$

Since μ, θ and α are constants, hence the fringe width is constant for monochromatic light. So the straight line fringes are equidistant

For normal incidence i= θ = 0 and the expression for fringe width becomes

$$\Delta t = \frac{\lambda}{2\mu \text{Cos}\alpha} \quad \text{................ ....(5.17)}$$

Which gives : $\boxed{\beta = \frac{\lambda}{2\mu\alpha \text{Cos}\alpha}}$ ....(5.18)

For an extremely thin film the angle of the wedge $\alpha \rightarrow 0$, and the fringe width becomes

$$\Delta t = \frac{\lambda}{2\mu} \quad \text{..(5.19)}$$

And $\boxed{\beta = \frac{\lambda}{2\mu\alpha}}$ ....(5.20)

Since μ and α are constants, hence the fringe width is constant for monochromatic light. So the straight line fringes are equidistant.

For a polychromatic light source

The refractive index 'μ' and angle of wedge 'α' is constant. So, for a particular order, 't' changes with wave length λ'. Hence for each wavelength of a polychromatic light a straight line pattern will be obtained for a given order. Hence corresponding to one order, a number of straight line fringes of different colours will be obtained as shown in Fig. (5.7).

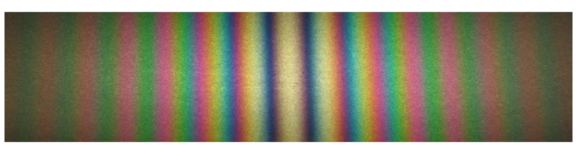
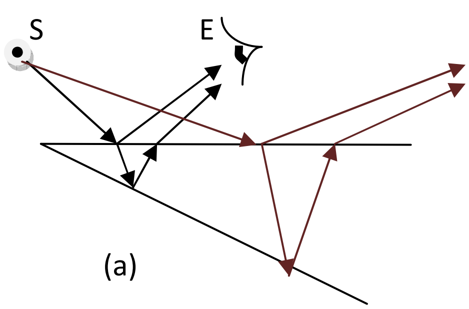


Fig. 5.8. Straight line coloured fringed formed by white light in wedge shaped film.

Need of an extended source



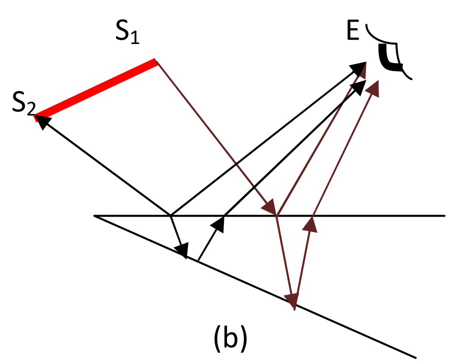


Fig. 5.9. Diagram demonstrating the need of extended source.

Let S be a narrow or point source of light as shown in Fig. (5.9a) A beam of light from S enters the eye E after one reflection from the upper surface of the film and one from lower surface. Another ray of light from S is incident at a different angle After the two reflections at the two surfaces of the film it cannot enter the eye as shown in Fig. (5.9a). So only a limited portion of the film become visible. To observe the whole film, the eye has to be continuously shifted from one position to the other.

If however an extended source S_1S_2 is used (fig. 5.9b) the light reflected from a large section of the film enters the eye, placed in a suitable position. Thus the entire film can be viewed simultaneously without moving the eye, as shown in (fig. 5.9b). Hence an extended source is required to view the film

Difference between fringes formed by Fizeau fringes and Haidinger fringes

Serial no.	Fizeau fringes	Haidinger fringes
1	These fringes are formed by interference due to a wedge shaped thin film.	These fringes are formed by interference due to a parallel sided to a thin film.
2	Fringes are loci of points of equal thickness.	Fringes are loci of points of equal inclination.
3	The fringes are straight lines parallel to the edge of the wedge.	The fringes are concentric circles
4	The fringes are localized.	The fringes are formed at infiniy
5	The zero order fringe is always present and is formed at the edge of the wedge whatever be the nature of the film and the medium on either side of it.	The zero order fringe is possible only if the medium on either side of the fringe is optically denser that the material of the film.
6	The fringe width is constant.	The angular fringe width decreases with decrease of order.
7	The fringe width is finite even for an ultra thin film and so the fringe pattern is present with different orders.	For an ultra thin film the angular fringe width tends to infinity and the zero order fringe fills up the entire field of view.

Industrial Application of thin film interference

Thin film interference can be used to test whether a surface is optically plane or not. A surface with no irregularities on it are known as **plane surface** and if the irregularities on the surface are of extremely smaller dimension compared to the wavelength of light, the surface is called **optically plane.**

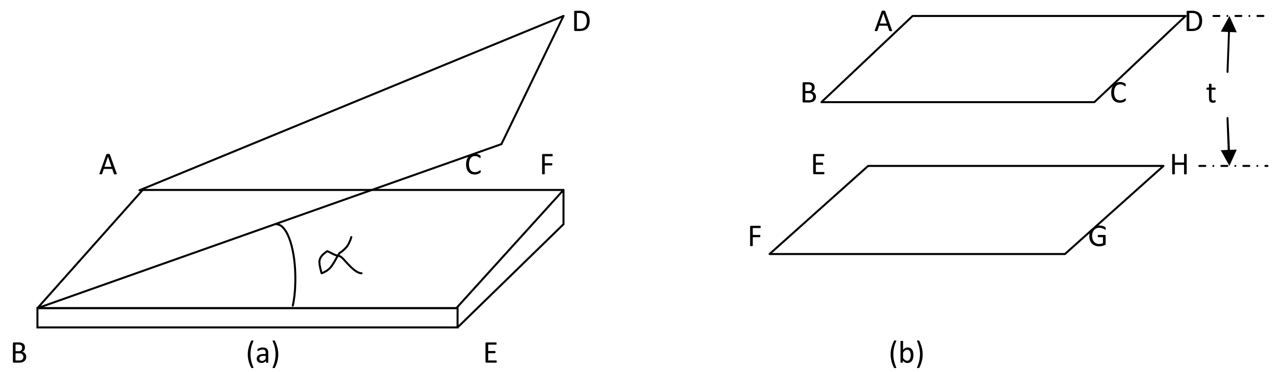


Fig. 5.10. Arrangement to test the optical planarity of a surface.

Let ABCD be the test plate. ABEF is another surface which is known to be optically plane. The test plate is placed in such a way that it makes an angle α with the plate ABEF so as to form a wedge shaped film with AB as edge (Fig. 5.10a). Monochromatic light is allowed to be incident on ABCD. If the fringes formed on ABCD are perfect straight lines then it can be inferred that ABCD is perfectly plane. If ABCD has irregularities on it, then the loci of points having constant thickness will no longer be a perfect a straight line. It will have a distorted appearance. So the plate ABCD ahs to be polished till equally placed perfect straight line fringes are obtained. Only then it can be inferred that the plate ABCD is optically plane.

Parallel sided thin film can also be used for this purpose. In this case the test plate ABCD is placed parallel to the known optically plane plate EFGH at a distance of ‘t’ from it (Fig. 5.10b). Monochromatic light is allowed to be incident on ABCD. If the fringes formed on ABCD are perfect concentric circles with fringe spacing decreasing outwards then it can be inferred that ABCD is perfectly plane. . If ABCD has irregularities on it, then the loci of points having constant inclination will no longer be a perfect circles. The circles will have a distorted appearance. So the plate ABCD ahs to be polished till equally placed perfect straight line fringes are obtained. Only then it can be inferred that the plate ABCD is optically plane.

Thin film interferometry can also be used to determine thickness of a thin plate, or the unknown wavelength of some light by using the expression for fringe width.

III. NEWTONS' RINGS.

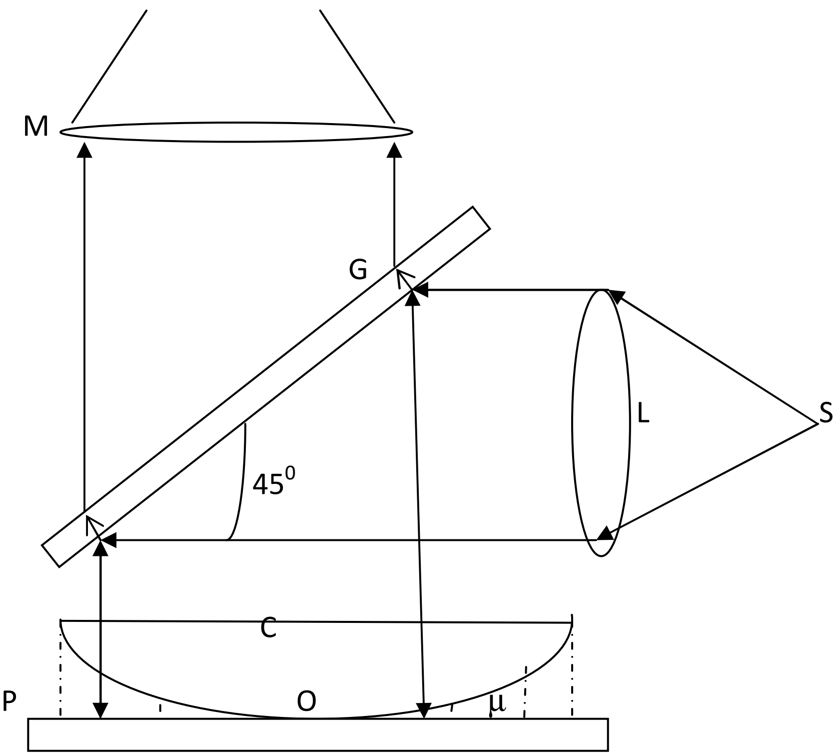


Fig. 5.11. The arrangement for formation of Newton's rings.

P $\rightarrow$ A plane glass plate

C $\rightarrow$ A plano convex lens of large radius of curvature placed on P

O $\rightarrow$ point where C touches P

C being of large radius of curvature it encloses a wedge shaped thin film of refractive index μ between itself and P. It is assumed that μ is less than the refractive index of P.

S $\rightarrow$ Source of light

L $\rightarrow$ A collimating lens.

G $\rightarrow$ A half silvered glass plate inclined at an angle of 45^0 to the horizontal

M$\rightarrow$ a microscope

Beams of light from S are rendered parallel by l and allowed to be incident on G. Being a half silvered plate G will reflect half of the incident light and transmit the remaining. As G is inclined at angle of 45^0 to the horizontal, hence the incident light will suffer 90^0 deviation on reflection and fall on C normally. As C has a large radius of curvature, it can be assumed that the light travels through C and is incident on the thin film normally. Consequently this light will be reflected from C and rereflected from P and finally gets transmitted through G to reach M. The light reflected from the upper and lower surfaces of the film, being coherent, interfere on superposition. Fringes of equal thickness are formed at the focal plane of M. Since the upper surface of the thin film is circular (concave) so the corresponding to a particular order the locus of points having constant thickness will be circular. Hence the fringes will be a set of bright and dark concentric circles with O as centre. These concentric circles or rings are called **Newton's rings.** At the point of contact, O the thickness of the film is zero. This corresponds to zero path difference and a zero order fringe. But due to reflection at the surface of denser medium, a phase change of π occurs, which leads the central fringe to be a dark fringe (Fig. 5.12). **Newtons' rings are also localized fringes as they are formed on the exit surface of the film.** Had the refractive index μ of the medium of the film been higher than that of P, there would have been no phase change of π and the zero order fringe would have been a bright one.

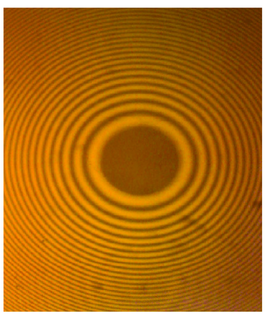


Fig. 5.12. Photograph of Newton's rings.

Conditions for maxima and minima.

The film concerned here is a wedge shaped thin film. The conditions of maxima and minima are given by:

$$2\mu t \mathrm{Cos}(\theta+\alpha) = (2n + 1)\frac{\lambda}{2} \qquad \text{for maxima}$$

$$= n\lambda \qquad \text{for minima}$$

As incidence is normal the angle 'θ' is zero. Due to large radius of curvature of the convex surface of the lens, the curved part tends to be a straight line and the angle of wedge α is also zero. This gives

$$\left.\begin{aligned} 2\mu t &= (2n + 1)\frac{\lambda}{2} \quad \text{for maxima} \\ &= n\lambda \quad \text{for minima} \end{aligned}\right\} \qquad \text{...(5.21)}$$

Calculation of radius of a Newton's rings

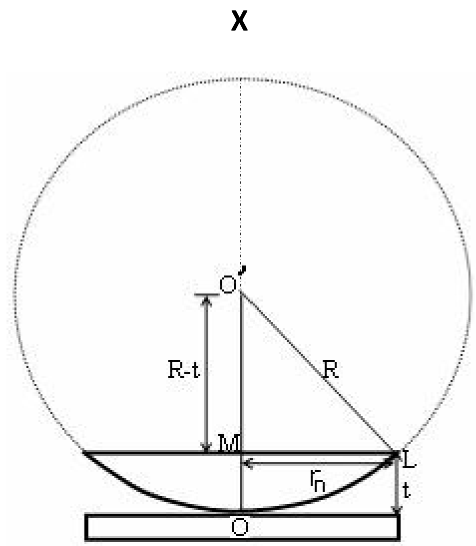


Fig. 5.13: Calculation of diameter of a ring of particular order.

$O^{/} \rightarrow$ centre of the circle of which the convex surface of the plano convex lens is a part.

$O^{/}L = O^{/}O = R \rightarrow$ Radius of curvature of the convex surface of the plano convex lens.

$XO = 2R \rightarrow$ diameter of curvature of the convex surface of the plano convex lens.

$ML = r_n \rightarrow$ Radius of the n^{th} order ring.

$O \rightarrow$ Point where the plano conves lens touched the glass plate

$MO = t \rightarrow$ thickness of film corresponding to order ring.

$O^{/}M = R – t$

From the geometry of the figure.

$$XM.MO = ML^2$$

Or $\quad (2R – t).t = r_n^2$

The film is a thin film and radius of curvature of the convex surface of the plano convex lens is high. So : $t << 2R$, which gives:

$$\mathbf{2Rt = r_n^2} \quad ……………………………………………….(5.22)$$

So $\quad r_n = \sqrt{2Rt} \quad …………………………………………….(5.23)$

To show that order of a fringe is directly proportional to the square of the radius

Substituting the expression of thickness 't' from equation to (5.22) to (5.21):

$$\left.\begin{aligned}\frac{\mu r_n^2}{R} &= (2n+1)\frac{\lambda}{2} \quad \text{for maxima} \\ &= n\lambda \text{ for minima}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.23)$$

Equation (5.23) shows that the order of the fringe is directly proportional to the square of its radius.

In terms of diameter D_n of the n^{th} fringe equation (5.23) becomes

$$\left.\begin{aligned}\frac{\mu D_n^2}{4R} &= (2n+1)\frac{\lambda}{2} \quad \text{for maxima} \\ &= n\lambda \text{ for minima}\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.24)$$

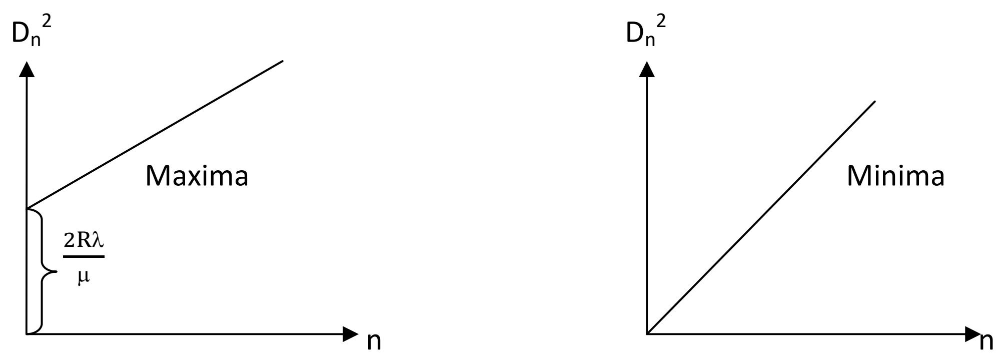


Fig. 5.14. Graph of D_n^2 vs. n.

If a graph of D_n^2 is plotted against order number 'n' then it is found that both for maxima and minima the slope of the graph is:

$$m' = \frac{4R\lambda}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.25)$$

For maxima the Y intercept of the graph is

$$Y_0 = \frac{2R\lambda}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.26)$$

The graph for monima passes through the origin.

Fringe width

Fringe width is the se[aration betweem successive maxima or successive minima. Hence fringe width 'β' will be the difference of radius betweem successive maxima or successive minima.

$$\beta = r_{m+1} - r_m \quad \text{...(5.27)}$$

Let r_m and r_{m+1} be the radii of the m^{th} and $(m+1)^{th}$ ring respectively. The corresponding diameters are Dm and $D_{m+1.}$ Using equation (5.23)

$$\frac{\mu r_p^2}{R} = m\lambda \qquad \text{for } m^{th} \text{ minima}$$

$$\frac{\mu r_{m+1}^2}{R} = (m+1)\lambda \qquad \text{for } (p+m)^{th} \text{ minima}$$

Subtracting one from the other

$$\frac{\mu(r_{m+1}^2 - r_m^2)}{R} = \lambda$$

Or $$\frac{\mu(r_{m+1} - r_m)(r_{m+1} + r_m)}{R} = \lambda$$

As the m^{th} and $(m+1)^{th}$ fringes are very close to each other hence: $r_{m+1} + r_m \approx 2r_m = D_m$

This gives $$\frac{\mu(r_{m+1} - r_m)D_m}{R} = \lambda$$

Or $$r_{m+1} - r_m = \frac{R\lambda}{\mu D_m}$$

Or $$\beta = \frac{R\lambda}{\mu D_m} \quad \text{...(5.28)}$$

The expression for fringe width shows that fringe width inversely proportional to the fringe diameter. So the fringe width decreases as ring diameter increases.

Measurement of unknown wavelength of light using Newton's rings.

The experimental arrangement shown in Fig. (5.13) is used with air film for which refractive index $\mu = 1$. The microscope M is focussed to visualise the Newton's rings. The microscope has a micrometer scale attached with it. By shifting the position of the microscope along the rings, the diameters D_p and D_{p+m} of the p^{th} and $(p+m)^{th}$ dark rings respectively are measured using the readings taken from the micrometer scale.

For p^{th} ring: $\frac{D_p^2}{4R} = p\lambda$ ……………………………….(5.29)

For $(p+m)^{th}$ ring: $\frac{D_{p+m}^2}{4R} = (p+m)\lambda$ …………………………..(5.30)

Substracting equation (5.29) from (5.30)

$$\frac{D_{p+m}^2 - D_p^2}{4R} = m\lambda$$

Or $$\lambda = \frac{D_{p+m}^2 - D_p^2}{4Rm} \quad \text{…………………………….(5.31)}$$

'm' can be obtained by counting the fringes. The radius of curvature 'R' of the plano convex lens can be measured by using a spherometer. Substituting all of them in equation (5.31) the value of wavelength 'λ' can be calculated.

<u>Graphical method:</u> The values of D_n for different values of 'n' are measured. D_n^2 is plotted against 'n' The slope of the graph '$m^/$' is calculated

$$m^/ = \frac{4R\lambda}{\mu} = 4R\lambda \quad \text{for air film}$$

Or $$\lambda = \frac{m^/}{4R} \quad . \quad \text{……………………………………(5.32)}$$

The radius of curvature 'R' of the plano convex lens can be measured by using a spherometer, which on substitution in equation (5.32) gives the value of λ. Use of bright fringes also gives the same result.

<u>Measurement of refractive index of a liquid using Newton's rings.</u>

The Newton's ring experiment is forst performed with air film between the plano convex lens and glass plate. The microscope M is focussed to visualise the Newton's rings. The microscope has a micrometer scale attached with it. By shifting the position of the microscope along the rings, the diameters D_p and D_{p+m} of the p^{th} and $(p+m)^{th}$ dark rings respectively are measured using the readings taken from the micrometer scale. As before we get

$$\frac{D_{p+m}^2 - D_p^2}{4R} = m\lambda$$

Or $$D_{p+m}^2 - D_p^2 = 4Rm\lambda \quad \text{…………………………………………………….(5.33)}$$

The experiment is again performed in a similar manner, after filling the space between the plano convex lens and glass plate with the liquid whose refractive index is to be measured. The film will now have a refractive index ‘μ’ Let the diameters of the p^{th} and $(p+m)^{th}$ ring be measured as before. Let D'_p and D'_{p+m} be the corresponding diameters. Hence

$$\frac{D'^2_{p+m} - D'^2_p}{4\mu R} = m\lambda$$

Or $$D'^2_{p+m} - D'^2_p = 4\mu R m\lambda \quad \text{...}(5.34)$$

From equation (5.33) and (5.34)

$$\mu = \frac{D'^2_{p+m} - D'^2_p}{D^2_{p+m} - D^2_p} \quad \text{..}(5.33)$$

However if λ is known one can use equation (5.34) to calculate μ

Effect of replacing the glass plate P with a mirror.

The planoconvex lens is not a very good reflector. It reflects a small portion of the incident light and transmits a major portion through the film to be incident on P. If P is a mirror instead of a glass plate, it will reflect almost the whole of the light incident on it. Consequently the light reflected from the lower surface of the film will have an amplitude much greter than the light reflected from the upper surface. So interference occurs between two coherent waves with widely different amplitudes and hence widely different intensities. For such an interference the minimas are very slightly less intense than the maximumm and the pattern will be indistinct.