## PHASE CHANGE ON REFLECTION: STOKES TREATMENT(Ref: Optics by A.B.Gupta)



Fig. 2.1a


Fig 2.1b

Fig 2.1a2.1) shows an absorption less surface $A B$ separating two media (1) and (2) It is assumed that medium (i) is optically rarer that medium (2) i.e. $\mu_{1}<\mu_{2}$, where. $\mu_{1}$ and $\mu_{2}$ are the refractive indices of (1) and (2)
$r=$ reflection coefficient of the surface of $A B$ facing (1)
$\mathrm{t}=$ transmission coefficient from (1) to (2)
$r^{\prime}=$ reflection coefficient of the surface of $A B$ facing (2)
$\mathrm{t}^{\prime}=$ transmission coefficient from (2) to (1)
A wave of amplitude ' $a$ ' is incident along PO at the point $O$ on $A B$. OR is the reflected wave and OQ is the transmitted wave.
ar = amplitude of OR
at = amplitude of $O Q$
Now keeping the point of incidence $O$ on $A B$ fixed let it be assumed that the directions of the waves OR and OQ are reversed along RO and QO respectively (Fig. 2.1b). Now, the principle of reversibility of light demands that their resultant effect should the incident light with same amplitude but reversed in direction i.e. along OP.

Now RO is reflected along OP and transmitted along OS
$\mathrm{ar}^{2}=$ amplitude of the reflected part of RO along OP
art = amplitude of the transmitted part of RO along OS
$\mathrm{atr}^{\prime}=$ amplitude of the reflected part of QO along OS
att $^{\prime}=$ amplitude of the transmitted part of QO along OP
So the net amplitude along OP is $=\mathrm{ar}^{2}+\mathrm{att}^{\prime}$.
According to our argument this amplitude should be ' $a$ '
Hence: $\quad a=a r^{2}+a t t^{\prime}$.
Or $\quad \mathrm{r}^{2}+\mathrm{tt}^{\prime}=1$
Also the net amplitude along OS is $=$ art + atr $^{\prime}$.
Since initially there was no wave along OS or SO, So OS will be non- existent. Consequently the amplitude of OS should be zero.

Hence: $\quad$ art + atr $^{\prime}=0$
Or $\quad r+r^{\prime}=0$
Or $\quad r^{\prime}=-r$
This shows that if one of the reflection coefficients is positive the other must be negative and vice versa. A negative refractive index denotes a phase change of $\pi$ of the wave on reflection. Hence a phase change of $\pi$ occurs on reflection either from rarer to denser medium or from denser to rarer medium, but from Stokes treatment it cannot be predicted which one.

The actual fact is revealed experimentally in Lloyd's mirror experiment, where interference occurs by the process of division of amplitude between waves direct from the source and waves reflected from an optically denser medium. It was found that the central fringe corresponding to zero path difference is dark instead of bright. This is possible only if light waves reflected from the surface of a denser medium undergoes a phase change of $\pi$. Thus a phase change of $\pi$ occurs when light coming from a rarer medium gets reflected at the surface of a denser medium.

## LLOYD'S MIRROR



Fig: 2.2. Schematic diagram of Lloyd's single mirror experiment.
$A B$ is a plane mirror. $S$ is a source of light. A beam of light $S A$ strikes the mirror at $A$ and gets reflected along AP. Another beam of light SB strikes the mirror at $B$ and gets reflected along $B Q$. Interference occurs at $P$ between the direct light from $S$ and the reflected light and interference fringes are obtained on a screen separated from the source by a distance ' $D$ '. AP and BQ appear to diverge from $S^{\prime}$, which is the virtual image of $S$. It seems that $S$ and $S^{\prime}$ are the two coherent sources of light. So out of the two coherent sources one is real and the other is virtual. Apart from the reflection mechanism, the scheme of interference is same as Young's double slit experiment and fringes of same shape will be formed. In the given fig. the point of zero path difference is outside the zone of interference. So, less than one half of the fringes, will be obtained within the region PQ. In order to observe the zero path difference fringe the screen has to be moved to the point $B$. At $B$ the light waves $S B$ and $S^{\prime} B$ have zero path difference and zero order fringe will be formed there. The zero order fringe was found to be a dark fringe. This indicates that even if the path difference is zero an additional phase difference of $\pi$ must have crept in from somewhere. SP being a direct beam from $S$ has no scope of phase change. So the only option is that the phase change must have occurred in the reflected ray. Here the incident ray SA after coming from a rarer medium (air) gets reflected at the mirror surface which is a denser medium. This clearly points out that when light is reflected at the surface of a denser medium it undergoes a phase change of $\pi$, and the corresponding reflection
coefficient is negative. Hence the reflection coefficient corresponding to reflection from a rarer surface is positive and in that case no phase change occurs.

## Determination of wavelength from Lloyd's mirror experiment

Comparing with Young's double slit experiment, here:
$D=$ distance between source and screen which is analogous to distance between double slit and screen.
$\frac{d}{2}=$ distance of real source $S$ from the mirror.
Since for reflection in a plane surface object distance is equal to the image distance hence:
$\frac{d}{2}=$ distance of virtual source $S^{\prime}$ from the mirror.
So the distance between the coherent sources is $=\mathrm{d}$
If $\lambda$ be the wavelength of light used then:
Fringe width $=\beta=\frac{D \lambda}{d}$
$\beta$ can be measured by taking readings. ' $D$ ' can be measured by mounting the screen and source on an optical bench. ' d ' can also be measured. Using these values $\lambda$ can be calculated from equation (2.3)

## FRESNEL BIPRISM.



Fig: 2.3: Schematic diagram of Fresnel Biprism

In Fresnel's biprism interference occurs by division of wavefront. The biprism BAC is a prism with the obtuse angle of prism $A \approx 179^{\circ}$. It can be considered to be a combination of two thin prisms ABD and ACD of very small angles with their bases facing each other and joined together.
$S \rightarrow A$ narrow source slit placed with its length parallel to the edge of the biprism, the edge being the line of intersection of the two faces represented by $A B$ and $A C$.

The incident wavefront is divided in two parts and suffer separate reflections from upper and lower prisms. The refracted beams being coherent interfere with each other and the resultant pattern is obtained on a screen. A beam of light SD incident normally on the biprism gets refracted along DQ due to the upper prism ABD and along DP due to the lower prism ACD after suffering a deviation $\delta$. Another two beams of light SB and SC get refracted along DR and DT due to the upper and lower prism respectively after suffering a deviation $-\delta$. The light beams within the cone $\mathrm{GS}_{1} T$ seem to diverge from $\mathrm{S}_{1}$ and those within the cone $\mathrm{RS}_{2} \mathrm{H}$ seem to diverge
from $S_{2}$. So $S_{1}$ and $S_{2}$ are the virtual images of $S$ which serve as coherent sources of light. Apart from the refraction mechanism, the scheme of interference is same as Young's double slit experiment and fringes of same shape will be formed.

## Determination of wavelength from biprism experiment

$\alpha \rightarrow$ Angle of each thin prism $-\angle A B D=\angle A C D$, where $\alpha$ is very small.
a $\rightarrow$ distance of $S$ from biprism,
b $\rightarrow$ distance of screen from biprism
$\delta \rightarrow$ angle of minimum deviation for each thin prism.
$\frac{d}{2}=$ Distance between $\mathrm{S}_{1}$ and $\mathrm{S}=$ Distance between $\mathrm{S}_{2}$ and S
So $d=$ Distance between $\mathrm{S}_{1}$ and $\mathrm{S}_{2}$
Comparing with Young's double slit experiment, here:
$D=a+b=$ distance between source and screen which is analogous to distance between double slit and screen.

The distance between the coherent sources is = d

If $\lambda$ be the wavelength of light used then the fringe width $\beta$ is given by

$$
\begin{equation*}
\beta=\frac{\mathrm{D} \lambda}{\mathrm{~d}}=\beta=\frac{(\mathrm{a}+\mathrm{b}) \lambda}{\mathrm{d}} \tag{2.4}
\end{equation*}
$$

$\beta$ can be measured by taking readings. ' $a$ ' and ' $b$ ' can be measured by mounting the screen and source on an optical bench. Hence if ' $d$ ' is known, then the unknown wavelength ' $\lambda$ ' of the light used can be calculated from equation (2.4). But ' $d$ ' being the distance between virtual sources can not be directly measured. It has to be measured using suitable experimental arrangements.

## Measurement of $\mathbf{d}$



Fig. 3.4. Experimental determination of $\beta$
$X Y$ is the focal plane of an eye piece. The source, biprism and an eye piece are mounted on an optical bench. A lens is placed between the biprism and eye piece, whose focal length is less than one fourth of the distance between them. The lens is moved parallel to the optical bench. There are two positions between the biprism and eye piece, where if the lens is placed real images of $S_{1}$ and $S_{2}$ are formed on XY. For position $L_{1}$, the image of $S_{1}$ is at $Q$ and that of $S_{2}$ is at $P$. Similarly for position $L_{2}$, the image of $S_{1}$ is at $C$ and that of $S_{2}$ is at $B$.
$\mathrm{d}_{1}=$ distance between P and Q
$d_{2}=$ distance between $B$ and $C$
Since the images are real hence $d_{1}$ and $d_{2}$ can be measured. The distance between $S_{1}$ and $S_{2}$ is given by $\quad d=\sqrt{d_{1} d_{2}}$ $\qquad$

## Determination of acute angle of biprism

The angle of minimum deviation $\delta$ being very small we can write

$$
\begin{align*}
& \delta=\tan \delta=\frac{\frac{d}{2}}{\mathrm{a}}=\frac{d}{2 a} \text {......................(2.6) From fig (2.3) } \\
& \text { So } \quad \mathrm{d}=2 \mathrm{a} \delta \text {.................................................7) }
\end{align*}
$$

If $\mu$ is the refractive index of the prism, then the expression for minimum deviation is given by:

$$
\delta=(\mu-1) \alpha \quad, \alpha \text { being the angle of prism. }
$$

Substituting this expression of $\delta$ in equation (2.7) we have

$$
\begin{align*}
d & =2 \mathrm{a}(\mu-1) \alpha \\
\text { Or } \quad \alpha & =\frac{d}{2 \mathrm{a}(\mu-1)} \tag{2.8}
\end{align*}
$$

' $d$ ' can be determined experimentally. ' $a$ ' can be measured. So with known value of $\mu$, the acute angle of biprism $(\alpha)$ can be calculated.

## Comparisn between biprism and Lloyds mirror

| Serial no. | Biprism fringes | Lloyd's mirror fringes. |
| :---: | :--- | :--- |
| 1 | In biprism the complete pattern of <br> fringes is obtained with zero order <br> fringe and higher orders on either <br> side of zero order. | Less than half of the fringe pattern is <br> obtained. The zero order is not obtained <br> unless the screen is in contact with one end <br> of mirror |
| 2 | Zero order fringe is bright | Zero order fringe is dark |
| 3 | The Zero order fringe is not very <br> sharp | The Zero order fringe if present is sharp |
| 4 | The two coherent sources producing <br> interference fringes are virtual. | One of the two coherent sources producing <br> interference fringes are real and the other <br> one is virtual. |

