COMPTON EFFECT

In 1924, Arthur Compton discovered that when a beam of monochromatic X-rays or γ rays (of shorter wavelength) are scattered by atoms of an element of low atomic number (such as graphite), the scattered radiation contains not only the original wavelength but also but also another new modified radiation of slightly longer wavelength – the **satellite line**. Scattering leading to a modified wavelength is called **incoherent scattering**. This type of incoherent X-Ray scattering is called **Compton effect.** The difference between the wavelength of original wave and the scattered modified wave is called **Compton shift.**

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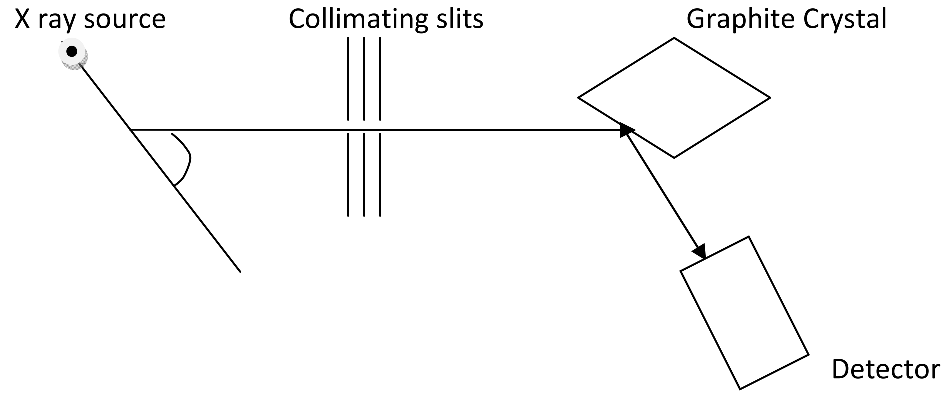

Fig. 3.1. Compton's experimental arrangement.

Characteristics of Compton shift

1. It is found that the Compton shift $d\lambda$ between the two radiations-primary and satellite varies with the angle of scattering.

2. The Compton shift $d\lambda$ is independent of the wavelength of the primary radiation.

3. The Compton shift $d\lambda$ is independent of the nature of the scatte

Explanation of Compton effect

The phenomenon of Compton effect could not be explained from the wave nature of radiation as it demands that scattered radiation should be of same wavelength as the incident one. But the result can be explained from the Quantum concept of radiation. The process is regarded as a collision between an incident photon and an electron of the scatterer keeping in mind that the conservation of energy and momentum holds good. If a photon of energy $h\nu$ strikes an electron it will impart some of its energy to the electron in the form of kinetic energy. Consequently the photon undergoes a loss of energy. To account for the decrease of energy, the scattered radiation should have a smaller frequency and hence a longer wavelength.

Mathematical approach and calculation of Compton shift.

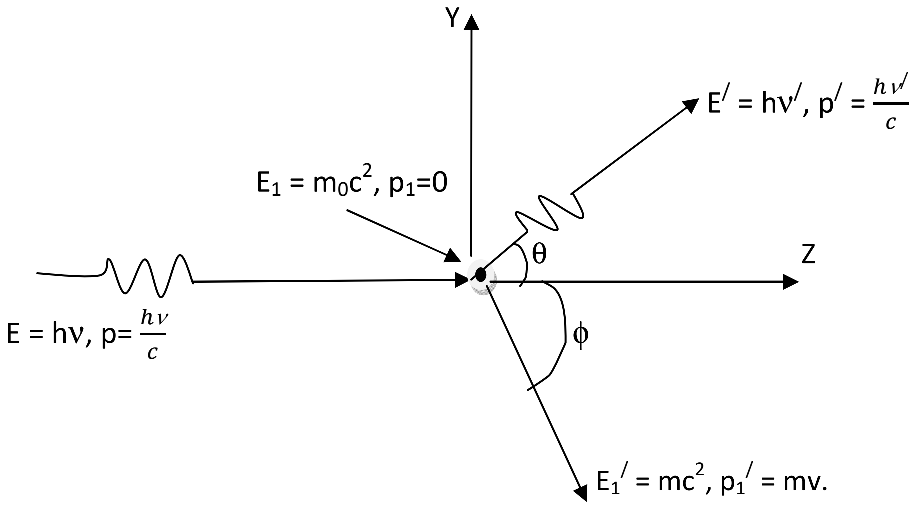

Fig. 3.2. Kinetics of Compton scattering.

Let a photon of energy hν collide with an electron of the target. Energy of photon before collision = E = hν and momentum before collision is $p = \frac{h\nu}{c}$, c, being the velocity of light in free space. After collision the electron recoils with a velocity 'v' in a direction making an angle ϕ with the direction of incident photon in clockwise sense, and a photon with lower energy is scattered in a direction making an angle θ with the direction of incidence in anticlockwise sense. The energy of the electron before scattering in $E_1 = m_0c^{2,}$ and its momentum is $p_1 = 0$, as the electron is initially at rest.

Total energy before collision is = $E_T = h\nu + m_0c^2$ ……………………(3.1)

Total momentum before collision = $p_T = \frac{h\nu}{c}$ ………………………………(3.2)

After collision:

Energy of recoil electron = $E_1^{/} = mc^2$ …………………………………(3.3) m is the relativistic mass.

Momentum of recoil electron = $p_1^{/} = mv$ ………………………………(3.4)

$P_1^{/}$ is in a direction making an angle ϕ with the direction of incident photon

Energy of scattered photon = $E^{/} = h\nu^{/}$ …………………………(3.5)

Momentum of scattered photon = $p^{/} = \frac{h\nu^{/}}{c}$ ……………………………(3.6)

$p^{/}$ is in a direction making an angle θ with the direction of incident photon.

Let Z be the direction of incidence and Y be the direction perpendicular to the direction of incidence.

Applying momentum conservation in Z direction

$$\frac{h\nu}{c} = \left(\frac{h\nu'}{c}\right)\text{Cos}\theta + mv\text{Cos}\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.7) \text{ from (3.2), (3.4) and (3.6)}$$

Applying momentum conservation in Y direction

$$0 = \left(\frac{h\nu'}{c}\right)\text{Sin}\theta - mv\text{Sin}\phi \quad \ldots\ldots\ldots\ldots\ldots\ldots (3.8) \text{ from (3.2), (3.4) and (3.6)}$$

Applying energy conservation

$$h\nu + m_0c^2 = h\nu' + mc^2 \ldots\ldots\ldots\ldots\ldots\ldots(3.9) \text{ from (3.1), (3.3) and (3.5)}$$

From (3.7) and (3.8)

$$mvc\text{Cos}\phi = h(\nu - \nu'\text{Cos}\theta) \ldots\ldots\ldots\ldots(3.10)$$

and $\quad mvc\text{Sin}\phi = h\nu'\text{Sin}\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots(3.11)$

Squaring and adding (3.10) and (3.11)

$$m^2v^2c^2 = h^2(\nu^2 + \nu'^2 - 2\nu\nu'\text{Cos}\theta) \quad \ldots\ldots\ldots(3.12)$$

from (3.9) $\quad mc^2 = m^0c^2 + h(\nu - \nu') \quad \ldots\ldots\ldots\ldots(3.13)$

squaring (3.13) $\quad m^2c^4 = m_0^2c^4 + h^2(\nu^2 + \nu'^2 - 2\nu\nu') + 2m_0c^2\, h(\nu - \nu') \quad \ldots\ldots\ldots(3.14)$

Subtracting (3.12) from (3.14)

$$m^2c^2(c^2 - v^2) = m_0^2c^4 - 2h^2\nu\nu'(1\text{-Cos}\theta) + 2m_0c^2\, h(\nu - \nu')$$

or $\quad m^2c^4\left(1 - \frac{v^2}{c^2}\right) = m_0^2c^4 - 2h^2\nu\nu'(1\text{-Cos}\theta) + 2m_0c^2\, h(\nu - \nu') \quad \ldots\ldots\ldots(3.15)$

Now the relativistic mass $m = \frac{m_0}{\sqrt{(1 - \frac{v^2}{c^2})}}$.

Hence $m_0 = m\sqrt{(1 - \frac{v^2}{c^2})}$ and $m_0^2 = m^2(1 - \frac{v^2}{c^2}) \quad \ldots\ldots\ldots\ldots(3.16)$

From (3.15) and (3.16)

$$m_0^2c^4 = m_0^2c^4 - 2h^2\nu\nu'(1\text{-Cos}\theta) + 2m_0c^2\, h(\nu - \nu')$$

or $0 = -2h^2\nu\nu^{/}(1\text{-Cos}\theta) + 2m_0c^2\, h(\nu - \nu^{/})$

or $2h^2\nu\nu^{/}(1\text{-Cos}\theta) = 2m_0c^2\, h(\nu - \nu^{/})$

or $h\nu\nu^{/}(1\text{-Cos}\theta = m_0c^2\,(\nu - \nu^{/})$

or $2h\nu\nu^{/}\text{Sin}^2\,\frac{\theta}{2} = m_0c^2\,(\nu - \nu^{/})$

or $2h\text{Sin}^2\,\frac{\theta}{2} = m_0c^2\left(\frac{\nu - \nu^{/}}{\nu\nu^{/}}\right)$

or $2h\text{Sin}^2\,\frac{\theta}{2} = m_0c^2\left(\frac{1}{\nu^{/}} - \frac{1}{\nu}\right)$

or $2h\text{Sin}^2\,\frac{\theta}{2} = m_0c\left(\frac{c}{\nu^{/}} - \frac{c}{\nu}\right)$

or $2h\text{Sin}^2\,\frac{\theta}{2} = m_0c(\lambda - \lambda^{/})$

or $2h\text{Sin}^2\left(\frac{\theta}{2}\right) = m_0c.d\lambda$, where $d\lambda = (\lambda - \lambda^{/})$ = Compton shift.

Or $d\lambda = \frac{2h\text{Sin}2\left(\frac{\theta}{2}\right)}{m_0c} = 2\lambda_0\ \text{Sin}^2\left(\frac{\theta}{2}\right)$ ……………………(3.17)

Equation (3.17) gives the expression for Compton shift.

$\lambda_0 = \frac{2h}{m_0c}$. m_0 = rest mass of electron, c = velocity of light and h = Planck's constant. All these quantities are universal constants. So λ_0 is also an universal constant which has a dimension of length and is called **Compton wavelength of an electron .** The value of λ_0 is 0.024Å. This shows that the Compton shift depends only on the angle of scattering but independent of the frequency of incident radiation and nature of target.

Energy of scattered photon.

Energy of scattered photon is $E^{/} = h\nu^{/}$

Now, it has been shown earlier that :

$$2h\text{Sin}^2\,\frac{\theta}{2} = m_0c^2\left(\frac{1}{\nu^{/}} - \frac{1}{\nu}\right)$$

Or $h(1 - \text{Cos}\theta) = m_0c^2\left(\frac{1}{\nu^{/}} - \frac{1}{\nu}\right)$

Or $\quad \frac{1}{\nu^{/}} = \frac{1}{\nu} + \frac{h(1-\text{Cos}\theta)}{m_0c^2}$

Hence $\nu^{/} = \frac{1}{\frac{1}{\nu}+\frac{h(1-\text{Cos}\theta)}{m_0c^2}}$

Or $\quad E^{/} = h\nu^{/} = \frac{h\nu}{1+\frac{h\nu(1-\text{Cos}\theta)}{m_0c^2}}$ = Energy of scattered photon in terms of scattering angle θ, and energy hν of incident photon.

Kinetic energy of recoil electron.

Since thethe kinetic energy is supplied by a part of the energy of the incident photon,hence gain in electron energy is equal to the loss in photon energy. So the kinetic energy of the recoil electron is obtained by simply subtracting the energy of the scattered photon from the energy of incident photon.

$$E_k = h\nu - \frac{h\nu}{1+\frac{h\nu(1-\text{Cos}\theta)}{m_0c^2}}$$

$$= h\nu\left(1-\frac{1}{1+\frac{h\nu(1-\text{Cos}\theta)}{m_0c^2}}\right)$$

$$E_k = h\nu\left(\frac{\frac{h\nu(1-\text{Cos}\theta)}{m_0c^2}}{1+\frac{h\nu(1-\text{Cos}\theta)}{m_0c^2}}\right)$$

$$= h\nu\left\{\frac{a(1-\text{Cos}\theta)}{1+a(1-\text{Cos}\theta)}\right\}$$

Where $a = \frac{h\nu}{m_0c^2}$

Relation between the angle of scattering of the photon 'θ', and the angle of deviation 'ϕ' of the recoil electron.

From equation (3.10) and (3.11):

$$\tan\phi = \frac{\nu^{/}Sin\theta}{\nu-\nu^{/}Cos\theta} \quad \text{...(3.18)}$$

Also from it has been deduced earlier that: $\frac{1}{\nu^{/}} = \frac{1}{\nu} + \frac{h\left(1-\text{Cos}\theta\right)}{m_0c^2}$

Or $$\frac{1}{\nu'} = \frac{1}{\nu}\left\{1 + \frac{h\nu(1 - \text{Cos}\theta)}{m_0c^2}\right\}$$

$$= \frac{1}{\nu}\left(1 + 2a\text{Sin}^2\left(\frac{\theta}{2}\right)\right) \quad \text{Where } a = \frac{h\nu}{m_0c^2}$$

Hence $$\nu' = \frac{\nu}{1+2a\text{Sin}^2\left(\frac{\theta}{2}\right)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.19)$$

Substituting the value of ν' from (3.19) in (3.18):

$$\tan\phi = \frac{\frac{\nu}{1+2a\text{Sin}^2\left(\frac{\theta}{2}\right)}Sin\theta}{\nu - \frac{\nu}{1+2a\text{Sin}^2\left(\frac{\theta}{2}\right)}Cos\theta}$$

$$= \frac{\frac{\nu}{1+2a\text{Sin}^2\left(\frac{\theta}{2}\right)}Sin\theta}{\frac{\nu(1-Cos\theta)+2a\nu\text{Sin}^2\left(\frac{\theta}{2}\right)}{1+2a\text{Sin}^2\left(\frac{\theta}{2}\right)}}$$

$$= \frac{\nu Sin\theta}{\nu\left\{((1-Cos\theta))+2a\text{Sin}^2\left(\frac{\theta}{2}\right)\right\}}$$

$$= \frac{2\nu Sin\left(\frac{\theta}{2}\right)Cos\left(\frac{\theta}{2}\right)}{2\nu\text{Sin}^2\left(\frac{\theta}{2}\right)\{1+a\}}$$

$$= \frac{Cot\left(\frac{\theta}{2}\right)}{1+a}$$

Or $$\text{Cot}\phi = tan\left(\frac{\theta}{2}\right)(1+a) = \left(1 + \frac{h\nu}{m_0c^2}\right) tan\left(\frac{\theta}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.20)$$

Equation (3.20) gives the Relation between the angle of scattering of the photon ‘θ’, and the angle of deviation ‘ϕ’ of the recoil electron.

Discussions

1. Compton shift depends only on the angle of scattering but independent of the frequency of incident radiation and nature of target.

2. The minimum Compton shift occurs when there is no scattering i.e. $\theta = 0$. The value of $(d\lambda)_{min} = 0$.

3. The maximum Compton shift occurs when the photon is scattered back i.e. $\theta = 180^0$. The value of $(d\lambda)_{max} = 2\lambda_0$

4. At $\theta = 90^0$, the value of $d\lambda = \frac{h}{m_0 c} = \lambda_0 = 0.024Å$, which is in agreement with the experimental results.

Comparisn between Compton scattering and photoelectric effect.

Serial no.	Compton scattering	Photoelectric effect
1	The energy of the incident light quantum should be greater than the binding energy of the electron, so that the electron recoils and a scattered photon of lower energy appears.	The energy of the incident light quantum should be of the same order of the binding energy of the electron. As the photon is absorbed, the binding energy is overcome by the acquired energy, and the electron as well as the resulting ion recoil with a certain energy and and momentum.
2	A part of the energy of the incident photon is absorbed by the electron.	The energy of incident photon is either completely absorbed by the electron or no energy is absorbed at all.