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Presentation
On
Classification of Signals and Systems

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Introduction to Signals

- Signals are variables that carry information.
- It is described as a function of one or more independent variables.
- Basically it is a physical quantity.It varies with some independent or dependent variables.
- Signals can be One-dimensional or multi-dimensional.

- **Signal:** *A function of one or more variables that convey information on the nature of a physical phenomenon.*

Examples: *v(t),i(t),x(t),heartbeat, blood pressure, temperature, vibration.*

- ***One-dimensional signals:*** *function depends on a single variable, e.g., speech signal*

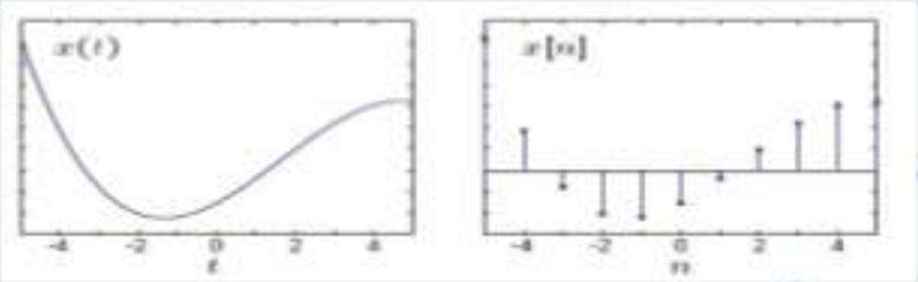
- ***Multi-dimensional signals:*** *function depends on two or more variables, e.g., image*

Classification of signals

- Continuous-time and discrete-time signals
- Periodic and non-periodic signals
- Casual and Non-casual signals
- Deterministic and random signals
- Even and odd signals

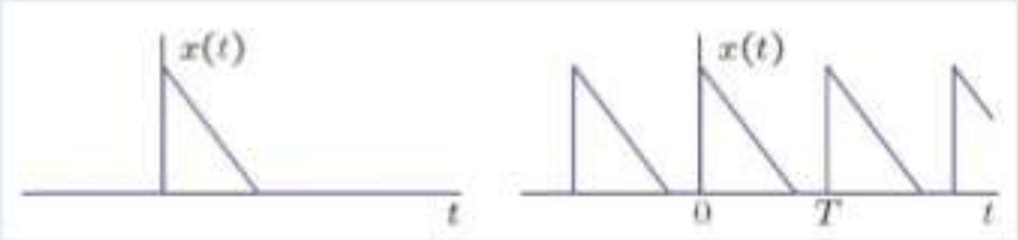
Continuous time (CT) & discrete time (DT) signals:

- CT signals take on real or complex values as a function of an independent variable that ranges over the real numbers and are denoted as x(t).
- DT signals take on real or complex values as a function of an independent variable that ranges over the integers and are denoted as x[n].
- Note the subtle use of parentheses and square brackets to distinguish between CT and DT signals.



Periodic & Non-periodic Signals

- Periodic signals have the property that $x(t + T) = x(t)$ for all t.
- The smallest value of T that satisfies the definition is called the period.
- Shown below are an non-periodic signal (left) and a periodic signal (right).

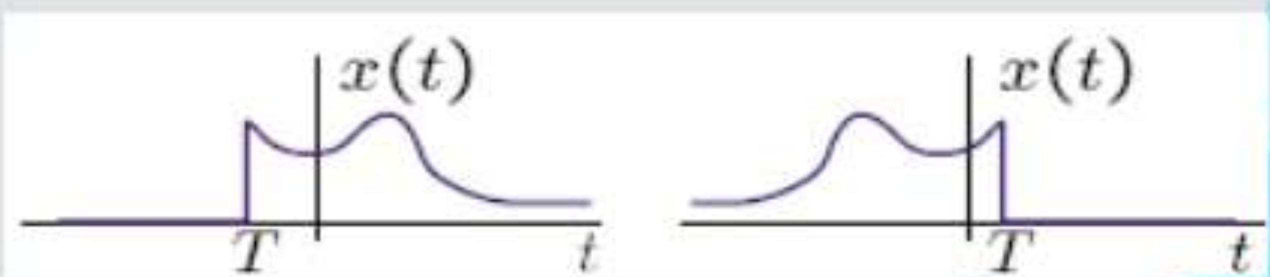


Causal &Non-causal Signals:

- A **causal** signal is zero for t < 0 and an **non-causal** signal is zero for t > 0

Right- and left-sided signals:

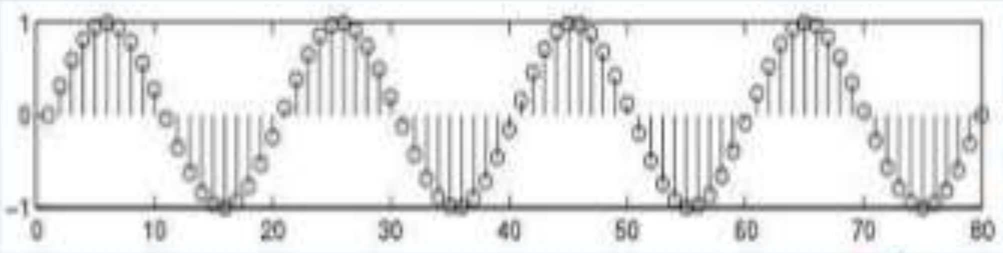
- A right-sided signal is zero for t < T and a left-sided signal is zero for t > T where T can be positive or negative.



Deterministic & Random Signals

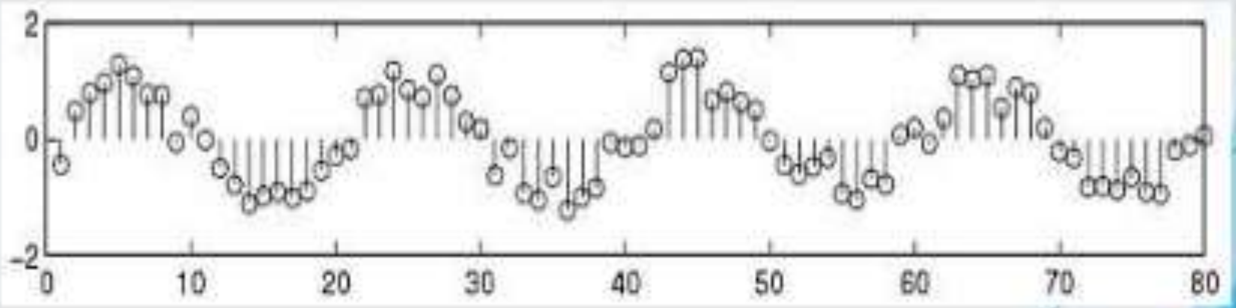
Deterministic signals :

- Behavior of these signals is predictable w.r.t time
- There is no uncertainty with respect to its value at any time.
- These signals can be expressed mathematically.
- For example $x(t) = \sin(3t)$ is deterministic signal.



Random Signals:

- Behavior of these signals is **random** i.e. not predictable w.r.t time.
- There is an uncertainty with respect to its value at any time.
- These signals can't be expressed mathematically.
- For example: **Thermal Noise** generated is non deterministic signal.



Even &Odd Signals

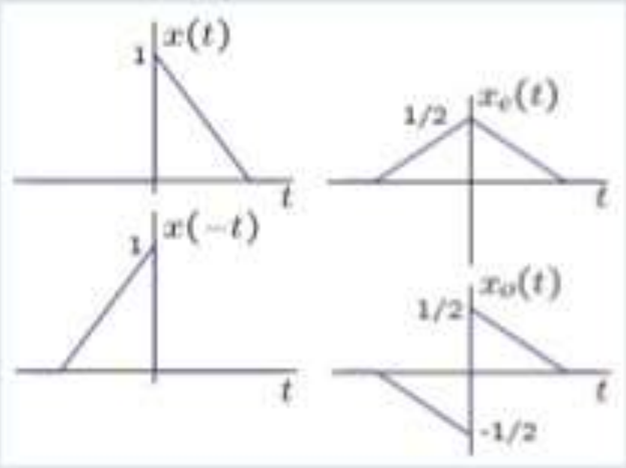
- Even signals xe(t) and odd signals xo(t) are defined as

$$x_e(t) = x_e(-t) \text{ and } x_o(t) = -x_o(-t).$$

- Any signal is a sum of unique odd and even signals. Using

$$x(t) = x_e(t)+x_o(t) \text{ and } x(-t) = x_e(t) - x_o(t), \text{ yields}$$

$$x_e(t) = 0.5(x(t)+x(-t)) \text{ and } x_o(t) = 0.5(x(t) - x(-t)).$$



Even & Odd Signals:

Even:

$$x(-t) = x(t)$$

$$x[-n] = x[n]$$

Odd:

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

- Any signal x(t) can be expressed as

$$x(t) = \ \ xe(t) + xo(t)\ )$$

$$x(-t) = xe(t) - xo(t)$$

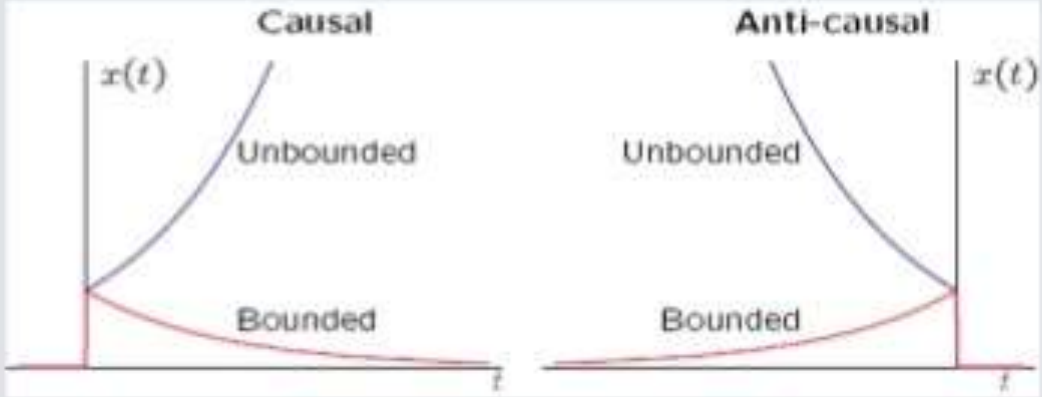
where

$$xe(t) = 1/2(x(t) + x(-t))$$

$$xo(t) = 1/2(x(t) - x(-t))$$

Bounded &Unbounded Signals:

- Every system is bounded, but meaningful signal is always bounded



Power and Energy Signals

- Power Signal
  - Infinite duration
  - Normalized power is finite and non-zero
  - Normalized energy averaged over infinite time is infinite
  - Mathematically tractable

- Energy Signal
  - Finite duration
  - Normalized energy is finite and non-zero
  - Normalized power averaged over infinite time is zero
  - Physically realizable

Elementary signals

- Step function
- Impulse function
- Ramp function

Unit Step function:

CT	DT
$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$	$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

Unit impulse function:

CT	DT
$\delta(t) = \begin{cases} 0, & t \neq 0 \\ \int_{-\infty}^{\infty} \delta(t)dt = \int_{0^-}^{0^+} \delta(t)dt = 1 \end{cases}$	$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$

Unit ramp function:

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$r(t) = tu(t)$$

$$u(t) = \frac{d}{dt} r(t) = \frac{d}{dt} t = 1$$

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

What is a System?

- Systems process input signals to produce output signals.

- **Examples:**
  - A circuit involving a capacitor can be viewed as a system that transforms the source voltage (signal) to the voltage (signal) across the capacitor
  - A CD player takes the signal on the CD and transforms it into a signal sent to the loud speaker
  - A communication system is generally composed of three sub-systems, the transmitter, the channel and the receiver. The channel typically attenuates and adds noise to the transmitted signal which must be processed by the receiver

How is a System Represented?

- A system takes a signal as an input and transforms it into another signal

- In a very broad sense, a system can be represented as the ratio of the output signal over the input signal
  - That way, when we "multiply" the system by the input signal, we get the output signal
  - This concept will be firmed up in the coming weeks

Types of Systems

- Causal & Non-causal
- Linear & Non Linear
- Time Variant &Time-invariant
- Stable & Unstable
- Static & Dynamic

Causal Systems

- Causal system : A system is said to be *causal* if the present value of the output signal depends only on the present and/or past values of the input signal.
- Example: $y[n]=x[n]+1/2x[n-1]$

Non-causal Systems

- Non-causal system : A system is said to be anticausal if the present value of the output signal depends only on the future values of the input signal.
- Example: $y[n]=x[n+1]+1/2x[n-1]$

Linear & Non Linear Systems

- A system is said to be linear if it satisfies the principle of superposition
- For checking the linearity of the given system, firstly we check the response due to linear combination of inputs
- Then we combine the two outputs linearly in the same manner as the inputs are combined and again total response is checked
- If response in step 2 and 3 are the same,the system is linear othewise it is non linear.

Time Invariant and Time Variant System

- A system is said to be *time invariant* if a time delay or time advance of the input signal leads to a identical time shift in the output signal.

$$y_i(t) = H\{x(t - t_0)\}$$
$$= H\{S^{t0}\{x(t)\}\} = HS^{t0}\{x(t)\}$$
$$y_0(t) = S^{t0}\{y(t)\}$$
$$= S^{t0}\{H\{x(t)\}\} = S^{t0}H\{x(t)\}$$

Linear Time-Invariant Systems

- Special importance for their mathematical tractability
- Most signal processing applications involve LTI systems
- LTI system can be completely characterized by their impulse response

$$y[n] = T\{x[n]\} = T\left\{\sum_{k=-\infty}^{\infty} x[k]\delta[n-k]\right\} \underline{\underline{Linearity}}$$

$$\sum_{k=-\infty}^{\infty} x[k]T\{\delta[n-k]\} = \sum_{k=-\infty}^{\infty} x[k]h_k[n] \underline{\underline{Time-Inv}}$$

$$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] = x[k] * h[k]$$

Stable & Unstable Systems

- A system is said to be *bounded-input bounded-output stable* (BIBO stable) iff every bounded input results in a bounded output.

i.e.

$$\forall t \quad |x(t)| \leq M_x < \infty \rightarrow \forall t \quad |y(t)| \leq M_y < \infty$$

Stable & Unstable Systems

Example: The system represented by

$y(t) = A\,x(t)$ is unstable ; $A>1$

Reason: let us assume $x(t) = u(t)$, then at every instant $u(t)$ will keep on multiplying with A and hence it will not be bonded.

Static Systems

- A static system is memoryless system
- It has no storage devices
- its output signal depends on present values of the input signal
- For example

$$i(t) = \frac{1}{R} v(t)$$

Dynamic Systems

- A dynamic system possesses memory
- It has the storage devices
- A system is said to possess *memory* if its output signal depends on past values and future values of the input signal

$$i(t) = \frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau$$

$$y[n] = x[n] + x[n-1]$$

Memoryless System

A system is memoryless if the output y[n] at every value of n depends only on the input x[n] at the same value of n

Example :

Square

$$y[n] = (x[n])^2$$

Sign

$$y[n] = \text{sign}\{x[n]\}$$

counter example:

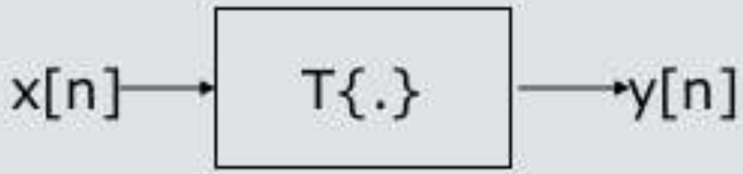
Ideal Delay System

$$y[n] = x[n - n_o]$$

Discrete-Time Systems

- Discrete-Time Sequence is a mathematical operation that maps a given input sequence x[n] into an output sequence y[n]

$$y[n] = T\{x[n]\}$$



- Example Discrete-Time Systems
  - Moving (Running) Average

$$y[n] = x[n] + x[n-1] + x[n-2] + x[n-3]$$

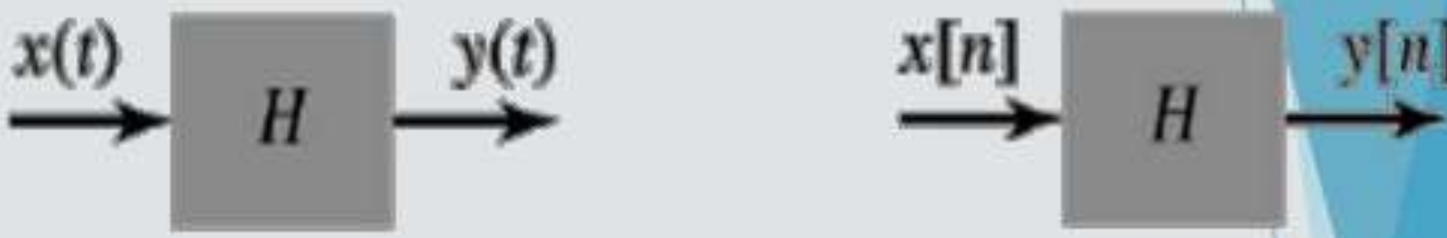
  - Maximum

$$y[n] = \max\{x[n], x[n-1], x[n-2]\}$$

  - Ideal Delay System

$$y[n] = x[n - n_o]$$

System Properties



(a) (b)

Notation: Let $\mathcal{H}$ represent the system, $x(t) \xrightarrow{\mathcal{H}} y(t)$ represent a system with input $x(t)$ and output $y(t)$.

1. Stability: BIBO (bounded input $\Rightarrow$ bounded output) stability

$$|x(t)| \leq M_x < \infty \implies |y(t)| \leq M_y < \infty$$

$$|x[n]| \leq M'_x < \infty \implies |y[n]| \leq M'_y < \infty$$

2.Memory /Memoryless:

- Memory system: present output value depend on future/past input.
- Memoryless system: present output value depend only on present input.
- Example:

Memory systems:

$$y(t) = 5x(t) + \int_{-\infty}^{t} x(\tau)d\tau$$

$$y[n] = \sum_{m=n-5}^{n+5} x[m]$$

Memoryless systems:

$$y[n] = x[n] + x^2[n]$$

3. Causal/noncausal

- Causal: present output depends on present/past values of input.
- Noncausal: present output depends on future values of input

Note: Memoryless ⇒ causal, but causal not necessarily be memoryless.

4. Time invariance (TI): time delay or advance of input ⇒ an identical time shift in the output.
   Let us define a system mapping $y(t) = \mathcal{H}(x(t))$. The system is time-invariant if

$$x(t - t_0) \xrightarrow{\mathcal{H}} y(t - t_0)$$

$$x[n - n_0] \xrightarrow{\mathcal{H}} y[n - n_0]$$

5. Linearity

Linear system: If $x_1(t) \xrightarrow{\mathcal{H}} y_1(t)$, $x_2(t) \xrightarrow{\mathcal{H}} y_2(t)$, then $ax_1(t) + bx_2(t) \xrightarrow{\mathcal{H}} ay_1(t) + by_2(t)$. Else, nonlinear.

- Superposition property (addition)
- Homogeneity (scaling)

Properties of a System:

- On this course, we shall be particularly interested in signals with certain properties:
- **Causal**: a system is causal if the output at a time, only depends on input values up to that time.
- **Linear**: a system is linear if the output of the scaled sum of two input signals is the equivalent scaled sum of outputs
- **Time-invariance**: a system is time invariant if the system's output is the same, given the same input signal, regardless of time.
- These properties define a large class of tractable, useful systems and will be further considered in the coming lectures

Thank You..!!