

# Communication

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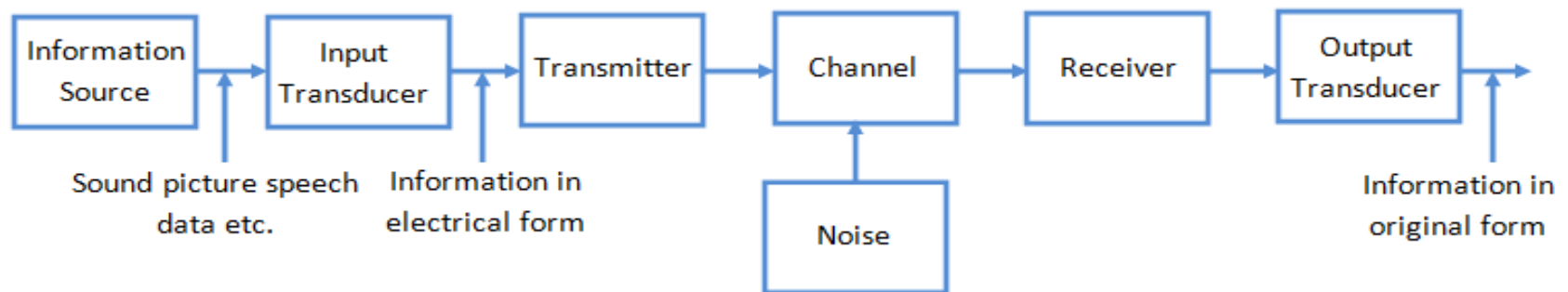
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# Communication

- ❑ Communication is the process of establishing connection or link between two points for information exchange.

In the most fundamental sense, communication involves the transmission of information from one point to another through a succession of process as listed below :

- 1.The generation of a thought pattern or image in the mind of an originator.
- 2.The description of that image, with a certain measure of precision, by a set of oral visual symbols.
- 3.The encoding of these symbols in a form that is suitable for transmission over a physical medium of interest.
- 4.The transmission of the encoded symbols to the desired destination.
- 5.The decoding and reproduction of the original symbols.
- 6.The recreation of the original thought pattern or image, with a definable degradation in quality, in the mind of recipient.



Block diagram of a general communication system

# Base-band communication

The base-band is used to designate the band of frequencies of the signal delivered by the source or the input transducer.

- In telephone, the base-band is the **audio band (band of voice signals)** of **0 to 3.5 kHz**.
  - In television, the base-band is video band occupying 0 to 4.3 MHz.
  - For digital data or PCM using bipolar signalling at a rate of  $f_0$  pulses/s, the base-band is 0 to  $f_0$  Hz.
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# Modulation

## Key Learning Points:

- **Terminology and Key Concepts of Modulation**
  - **Basic Types of Modulation**
  - **Pulse Shaping Techniques**
  - **Geometric Representation**
  - **Linear (QPSK) and Constant Envelope (MSK) Techniques**
  - **Combined Techniques (QAM)**
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# Why Modulation?

- Ease of radiation
- The size of antenna  $\propto \lambda/4 = c/4f$ 
  - If we wish to throw a piece of paper (base-band signal), it cannot go too far by itself. But by wrapping it around a stone(carrier), it can be thrown over a longer distance.
- Simultaneous transmission of several signals
- FDM (Frequency Division Modulation)
- Reduce the influence of interference

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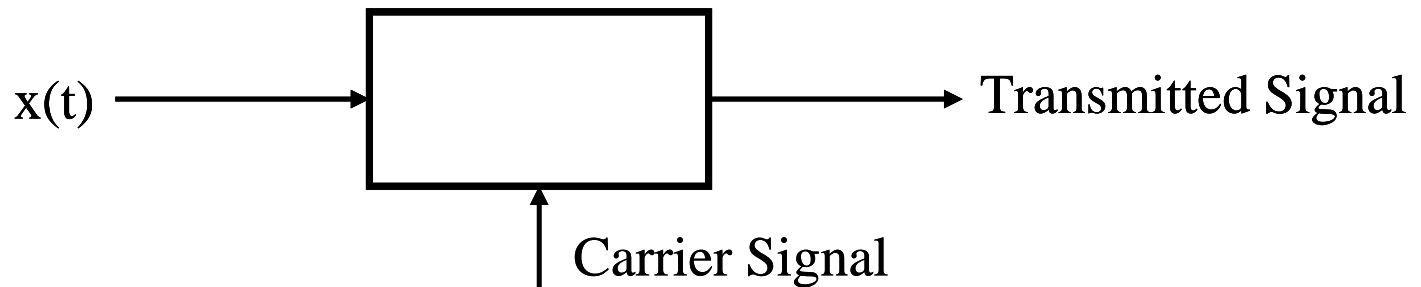
- Frequency Hopping
- Effecting the exchange of SNR with B
- Shannon's equation :  $C = B \log_2(1 + SNR)$ 
  - C is rate of information change per second (bit/s)

# The Concept of Modulation

**Modulation** ≡

process of varying carrier wave in accordance with bearing signal information (message signal).

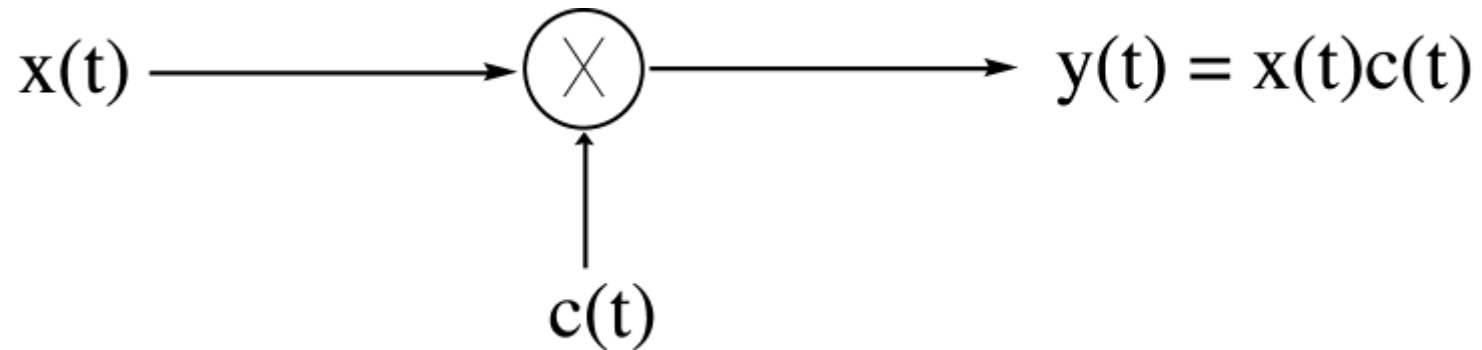
- i. **shift spectral content** of data signal to lie in a channel;
- ii. Efficient transmission of message signals at **higher frequencies**;
- iii. put information signal in a form that has **more noise immunity**;
- iv. permit **multiple access** to channel with variety of techniques;  
(Transmitting **multiple signals** through the same medium using different carriers).
- v. Transmitting through “**channels**” with limited pass-bands.
- vi. Others.....



# The Concept of Modulation

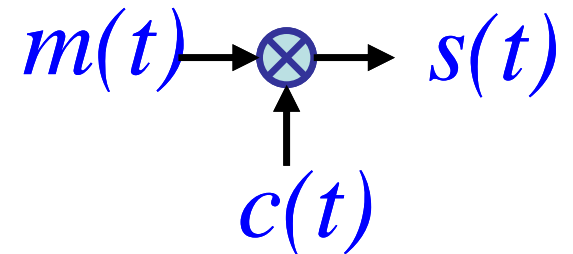
How?

- *Many* methods



## Linear & Non Linear Modulation

- $m(t)$  = modulating signal (data)
- $c(t)$  = sinusoidal carrier
- $s(t)$  = modulated signal



# The Concept of Modulation

Carrier  $c(t)$  given as  $c(t) = A_c \cos(2\pi f_c t + \theta)$

vary parameters of  $c(t)$  based on  $m(t)$  to produce  $s(t)$

(1) amplitude modulation  $A_c$  is varied linearly.

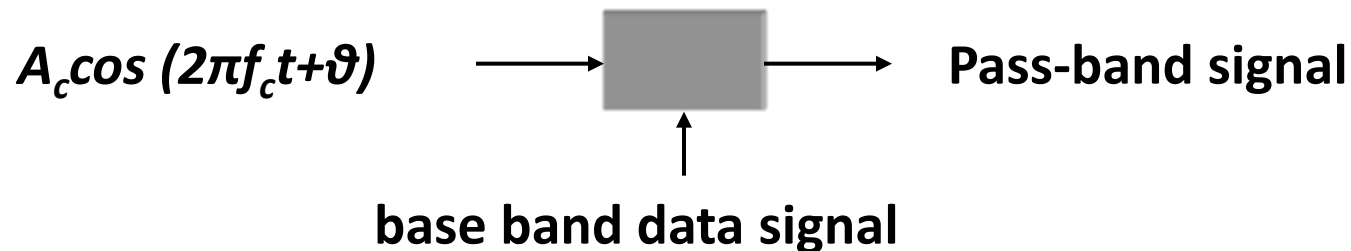
(2) angle modulation – carrier angle  $\varphi$  is varied linearly

$$\varphi = 2\pi f_c t + \theta$$

- frequency  $f_c$  is varies linearly with  $m(t)$

- phase  $\vartheta$  is varies linearly with  $m(t)$

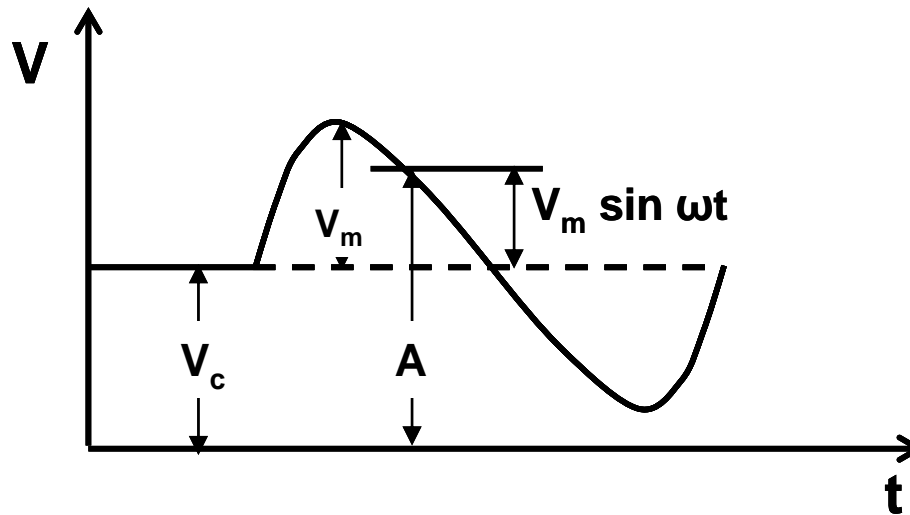
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# Amplitude Modulation

Let us consider the following diagram:



$$A = V_c + v_m = V_c + V_m \sin \omega_m t = V_c + mV_c \sin \omega_m t = V_c (1 + m \sin \omega_m t)$$

where,  $m = \frac{V_m}{V_c}$  is called the modulation index.

The instantaneous voltage of the resulting amplitude-modulated wave is,  
 $v = A \sin \theta = A \sin \omega_c t = V_c (1 + m \sin \omega_m t) \sin \omega_c t$

# Amplitude Modulation (continued..)

Look at the following trigonometric relation,

$$\sin x \sin y = \frac{1}{2} [\cos(x - y) - \cos(x + y)]$$

Then we can write the amplitude modulated wave as:

$$v = V_c \sin \omega_c t + \frac{mV_c}{2} \cos(\omega_c - \omega_m)t - \frac{mV_c}{2} \cos(\omega_c + \omega_m)t$$

## What do we see?

- An amplitude-modulated wave contains three terms:

- ✓ The un-modulated carrier.

- ✓ The two additional terms (*called two sidebands*)

- frequency of the lower sideband (LSB) is,

$$f_c - f_m$$

- frequency of the upper sideband (USB)

$$f_c + f_m$$

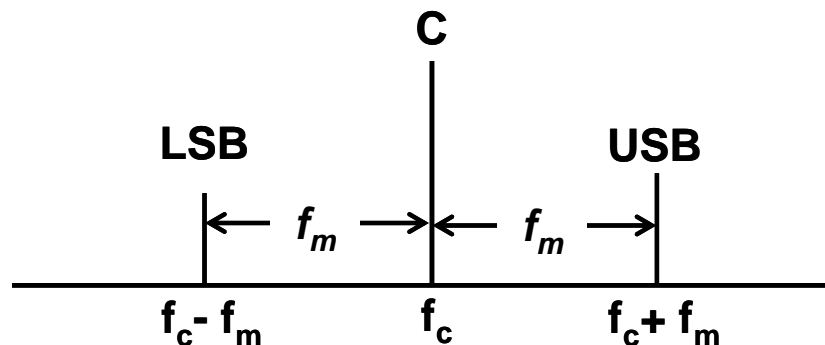
- **Bandwidth:**

- twice the frequency of the modulating signal.  $(\omega_c - \omega_{\max})$  to  $(\omega_c + \omega_{\max})$

- modulation by several sine waves (as in the AM broadcasting service): twice the highest modulating frequency.

# Representation of the modulated wave

- Amplitude modulation may be represented in any of three ways, depending on the *point of view*.



- The figure shows the spectrum of a modulating wave consisting of three discrete frequencies.
- Central frequency (carrier) has the highest amplitude, and the other two are disposed symmetrically about it having equal amplitudes but can never exceed half the carrier amplitude as has been seen from the modulated wave equation:

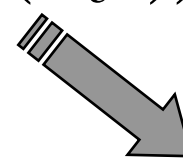
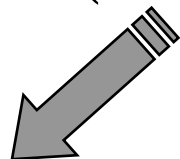
$$v = V_c \sin \omega_c t + \frac{mV_c}{2} \cos(\omega_c - \omega_m)t - \frac{mV_c}{2} \cos(\omega_c + \omega_m)t \quad \text{and } m > 1$$

# AM – Fourier Domain

$$s(t) = A_c (1 + k_a s_m(t)) \cos(\omega_c t)$$

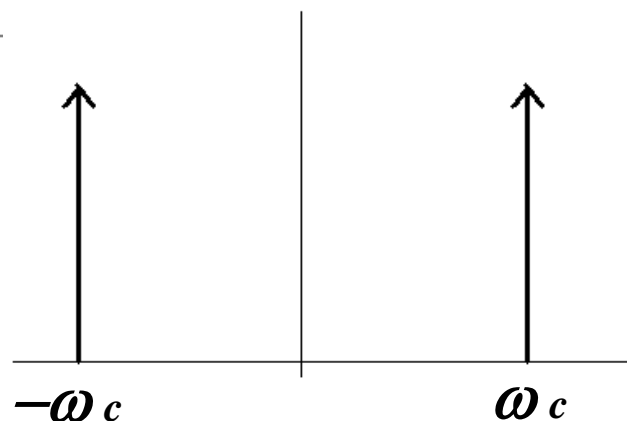
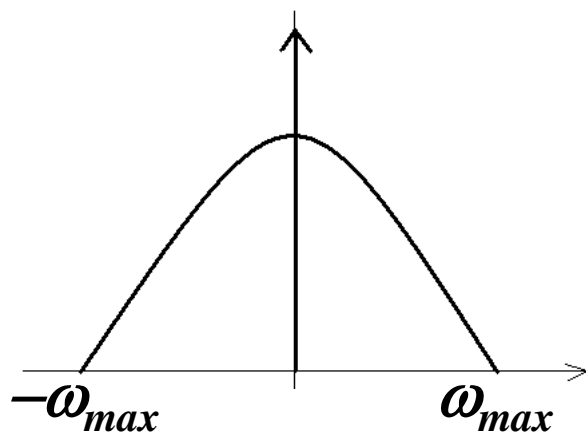


$$S(\omega) = A_c \mathfrak{F}(1 + k_a s_m(t)) * \mathfrak{F}(\cos(\omega_c t))$$

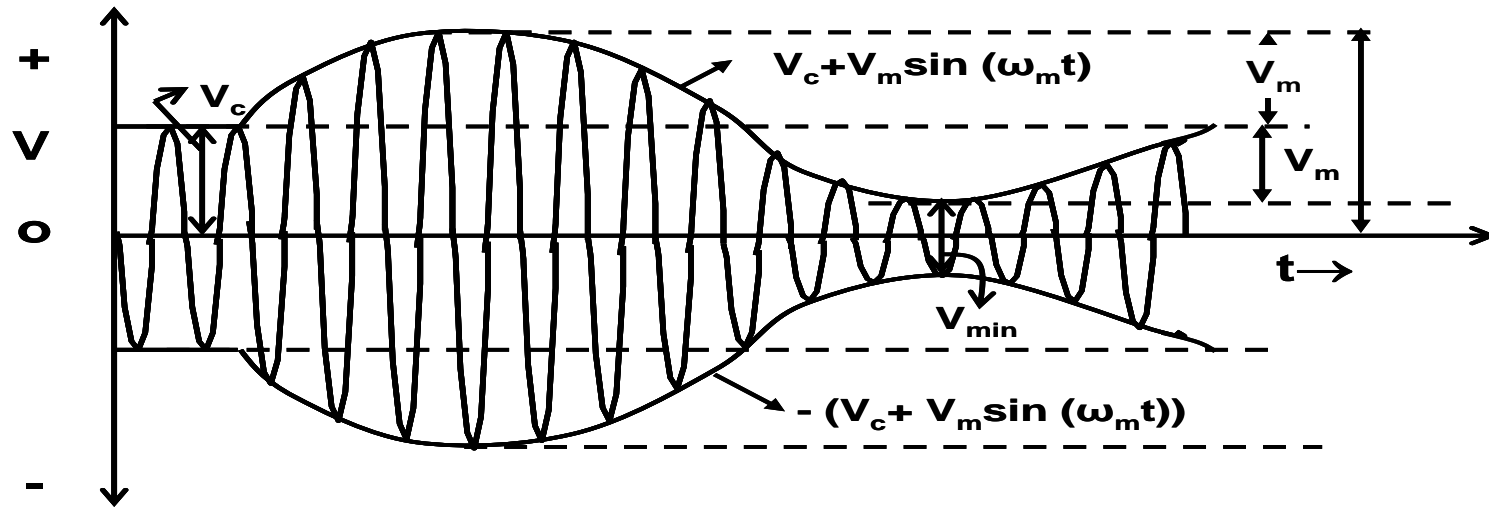


$$\delta(\omega) + k_a S_m(\omega)$$

$$\pi(\delta(\omega - \omega_c) + \delta(\omega + \omega_c))$$



# Amplitude modulation index



$$V_m = \frac{V_{\max} - V_{\min}}{2}$$

$$V_c = V_{\max} - V_m = V_{\max} - \frac{V_{\max} - V_{\min}}{2} = \frac{V_{\max} + V_{\min}}{2}$$

$$m = \frac{V_m}{V_c} = \frac{\frac{V_{\max} - V_{\min}}{2}}{\frac{V_{\max} + V_{\min}}{2}} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$$

# Amplitude modulation index

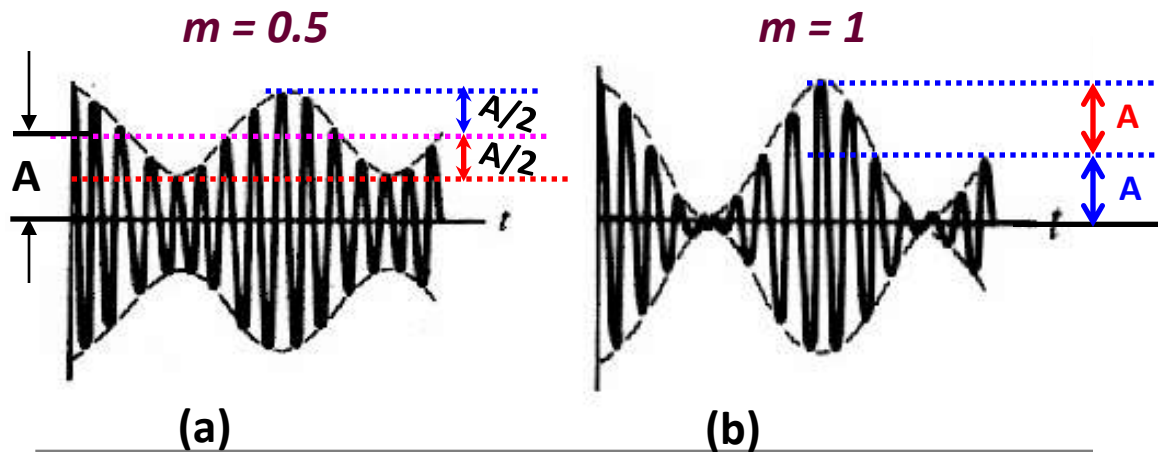
Equation  $m = \frac{V_m}{V_c} = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}}$  is the standard method of evaluating

modulation index when calculating from a waveform.

- It can be measured from an oscilloscope.
  - when both the carrier and the modulating voltages are known.
- When only the RMS values of the carrier and the modulated voltage or current are known, or when the un-modulated and the modulated output powers are given, it is necessary to and use the power relations in the AM wave.
- Finally, if the main interest is the instantaneous modulated voltage, the phasor diagrams depicting the three individual components of the AM wave may be drawn.

# AM Modulation

Sketch the envelop of the modulated waves for  $m=0.5$  and  $m=1$ , when  $m(t) = A_m \cdot \cos(\omega_m t)$



# AM – Percentage Modulation (Definitions)

✓ The percentage modulation of an AM signal is

$$\% \text{ Modulation} = \frac{m(t)_{max}}{A_c} \times 100 \%$$

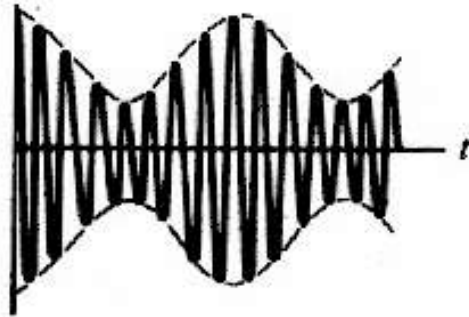
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➤ The Transmission efficiency of an AM signal is defined as

$$\eta = \frac{\text{Useful Power}}{\text{Total Power}} = \frac{P_S}{P_t}$$



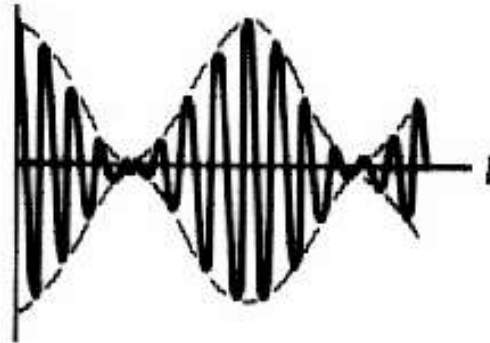
# AM – Percentage Modulation



**Under modulated (<100%)**



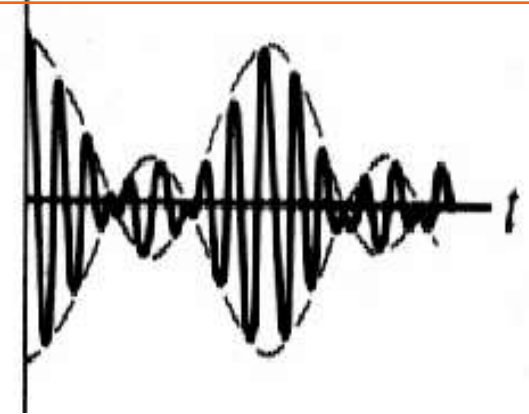
Envelope Detector  
Can be used



**100% modulated**



Envelope Detector  
Gives Distorted signal



**Over Modulated (>100%)**



# AM – Normalized Average Power

The normalized average power of the AM signal is:

$$\begin{aligned}\langle s^2(t) \rangle &= \frac{1}{2} \langle |g(t)|^2 \rangle = \frac{1}{2} A_c^2 \langle [1 + m(t)]^2 \rangle \\ &= \frac{1}{2} A_c^2 \langle [1 + 2m(t) + m^2(t)] \rangle \\ &= \frac{1}{2} A_c^2 + A_c^2 \langle m(t) \rangle + \frac{1}{2} A_c^2 \langle m^2(t) \rangle\end{aligned}$$

If the modulation contains no dc level, then  $\langle m(t) \rangle = 0$

The normalized power of the AM signal is:

$$\langle s^2(t) \rangle = \underbrace{\frac{1}{2} A_c^2}_{\text{Discrete carrier power}} + \underbrace{\frac{1}{2} A_c^2 \langle m^2(t) \rangle}_{\text{Sideband power}}$$

# Power relation in Amplitude Modulation

The total power in the modulated wave will be:

$$P_t = \frac{V_{carr}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

$$P_c = \frac{V_{carr}^2}{R} = \frac{\left(\frac{V_c}{\sqrt{2}}\right)^2}{R} = \frac{V_c^2}{2R}$$

$$P_{LSB} = P_{USB} = \frac{V_{SB}^2}{R} = \frac{\left(\frac{mV_c}{\sqrt{2}}\right)^2}{R} = \frac{m^2 V_c^2}{8R} = \frac{m^2 V_c^2}{4 \cdot 2R}$$

where all three voltages are rms values, and R is the resistance (e.g., antenna resistance) in which the power is dissipated. The first term of the above equation is the un-modulated carrier power and is given by:

The total power in the modulated wave will be:

$$P_t = \frac{V_c^2}{2R} + \frac{m^2 V_c^2}{4 \cdot 2R} + \frac{m^2 V_c^2}{4 \cdot 2R} = P_c + \frac{m^2}{4} P_c + \frac{m^2}{4} P_c$$

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2}$$

# Power: sideband and carrier

Total power of the AM signal is:

$$P_t = P_c + P_s = \frac{1}{2} \left[ A_c^2 + A_c^2 \langle m^2(t) \rangle \right] = \frac{A_c^2}{2} \left( 1 + \langle m^2(t) \rangle \right)$$

The % of the total power carried out by the sidebands is  $\eta$ , given by

$$\eta = \frac{P_s}{P_t} \times 100\% = \frac{\langle m^2(t) \rangle}{A_c^2 + \langle m^2(t) \rangle} \times 100\%$$

For the special case of tone modulation,

$$m(t) = mA_c \cos \omega_m t \quad \text{and} \quad \langle m^2(t) \rangle = \frac{m^2 A_c^2}{2}$$

Thus, 
$$\eta = \frac{m^2}{2 + m^2} \times 100\% \quad \text{with the condition that } m \leq 1.$$

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It can be seen that  $\eta_{\max}$  occurs at  $m=1$  and is given by,  $\eta_{\max}=33\%$ .

For the tone modulation, the maximum efficiency is 33%. This means that under the best conditions, only a third of the transmitted power carries the information. Moreover, this efficiency is for tone modulation. *For voice signals it is even worse which is ~ 25%.*

**Volume compression and peak limiting is commonly used in AM to ensure that full modulation is maintained most of the time.**

# Problem

**Determine  $\eta$  and the percentage of the total power carried out by the sidebands of the AM wave for tone modulation when (1)  $m = 0.5$  and (2)  $m = 0.3$ .**

We know that, 
$$\eta = \frac{m^2}{2 + m^2} \times 100\%$$

For  $m=0.5$ , 
$$m = 0.5$$

$$\eta = \frac{0.25}{2.25} \times 100\% = 11.11\%$$

- **Hence, only 11% of the total power is in the sidebands.**
- 

For  $m=0.3$ , 
$$m = 0.3$$

$$\eta = \frac{0.09}{2.09} \times 100\% = 4.3\%$$

- **Thus, only 4.3% of the total power is the useful power (i.e., power in the sidebands).**

# Modulation index from current

If the modulated and un-modulated currents are easily measurable, then the modulation index can be calculated from them.

Let  $I_c$  be the un-modulated current and  $I_t$  the total, or modulated current of an AM transmitter, both being rms values. If  $R$  is the resistance in which these currents flow, then

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left( \frac{I_t}{I_c} \right)^2 = 1 + \frac{m^2}{2}$$

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$$\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}$$

$$I_t = I_c \sqrt{1 + \frac{m^2}{2}}$$

# Notes: Power relation in AM

- Power relation:  $\frac{P_t}{P_c} = 1 + \frac{m^2}{2}$
- Relates the total power in the amplitude-modulated wave to the unmodulated carrier power.
- This must be used to determine, among other quantities, the modulation index in instances.
- It is interesting to note from the equation that the maximum power in the AM wave is
$$P_t = 1.5P_c \text{ when } m = 1.$$
- This is important, because it is the maximum power that the relevant amplifiers must be capable of handling without distortion.

# Problem: Power in AM

1. A 400-watt (400-W) carrier is modulated to a depth of 75%. Calculate the total power in the modulated wave.

Solution:

$$\frac{P_t}{P_c} = 1 + \frac{m^2}{2} = 400 \left( 1 + \frac{0.75^2}{2} \right) = 400 \times 1.281 = 512.5W$$

2. A broadcast radio transmitter radiates 10 kilowatts (10 kW) when the modulation percentage is 60. How much of this is carrier power?
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Solution:

$$P_c = \frac{P_t}{1 + \frac{m^2}{2}} = \frac{10}{1 + \frac{0.6^2}{2}} = \frac{10}{1.18} = 8.47kW$$



# Problem: Power in AM

*The antenna current of an AM transmitter is 8 amperes (8 A) when only the carrier is sent, but it increases to 8.93 A when the carrier is modulated by a single sine wave. Find the percentage modulation. Determine the antenna current when the depth of modulation changes to 0.8.*

**Solution:** 
$$\left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m^2}{2}; \quad \frac{m^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1; \quad m = \sqrt{2 \times \left[ \left(\frac{I_t}{I_c}\right)^2 - 1 \right]}$$

**Here,** 
$$m = \sqrt{2 \times \left[ \left(\frac{8.93}{8}\right)^2 - 1 \right]} = \sqrt{2 \times [(1.116)^2 - 1]}$$

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$$= \sqrt{2 \times (1.246 - 1)}$$
$$= \sqrt{0.492}$$
$$= 0.701 = 70.1\%$$

# Modulation by several sine waves

Total power in an AM wave consists of carrier power and sidebands' power.

This yields: 
$$P_t = P_c \left(1 + \frac{m^2}{2}\right) = P_c + \frac{P_c \cdot m^2}{2} = P_c + P_{SB} \quad \text{where, } P_{SB} = \frac{P_c \cdot m^2}{2}$$

If several sine waves simultaneously modulate the carrier, the carrier power will be unaffected, but the total sideband power will now be the sum of the individual sidebands' power. Hence we have,

$$P_{SB_T} = P_{SB_1} + P_{SB_2} + P_{SB_3} + \dots$$

$$\frac{P_c m_t^2}{2} = \frac{P_c m_1^2}{2} + \frac{P_c m_2^2}{2} + \frac{P_c m_3^2}{2} + \dots$$

$$m_t^2 = m_1^2 + m_2^2 + m_3^2 + \dots$$

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \dots}$$

# Points to be noted:

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- Two approaches yield the same result
  - Total modulation index = square root of sum of the squares of individual modulation indices.
  - Total modulation index must still not exceed unity, or distortion will result in with over- modulation by a single sine wave.
  - Whether modulation is by one or many sine waves, the output of the modulated amplifier will be zero for the negative modulating voltage peak if over- modulation takes place.
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# Problem

***A certain transmitter radiates 9 kW with the carrier un-modulated and 10.125 kW when the carrier is sinusoidally modulated. Calculate the modulation index. If another sine wave, corresponding to 40 percent modulation, is transmitted simultaneously, determine the total radiated power.***

**Solution:**

$$\frac{m^2}{2} = \frac{P_t}{P_c} - 1 = \frac{10.125}{9} - 1 = 1.125 - 1 = 0.125$$

$$m^2 = 0.125 \times 2 = 0.25$$

$$m = \sqrt{0.25} = 0.5$$

**For the second part, the total modulation index will be**

$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{0.5^2 + 0.4^2} = \sqrt{0.25 + 0.16} = \sqrt{0.41} = 0.64$$

$$P_t = P_c \left(1 + \frac{m_t^2}{2}\right) = 9 \times \left(1 + \frac{0.64}{2}\right) = 9 \times (1 + 0.205) = 10.84 \text{ kW}$$

# Problem

*The antenna current of an AM broadcast transmitter, modulated to a depth of 40 percent by an audio sine wave, is 11 A. It increases to 12 A as a result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?*

**Solution:**

$$I_c = \frac{I_T}{\sqrt{1 + \frac{m^2}{2}}} = \frac{11}{\sqrt{1 + \frac{0.4^2}{2}}} = \frac{11}{\sqrt{1 + 0.08}} = 10.58\text{A}$$

**Total modulation index  $m_t$**

$$m_t = \sqrt{2 \times \left[ \left( \frac{I_T}{I_c} \right)^2 - 1 \right]} = \sqrt{2 \times \left[ \left( \frac{12}{10.58} \right)^2 - 1 \right]} = \sqrt{2 \times (1.286 - 1)} = 0.757$$

Using equation  $m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \dots}$  we obtain,

$$m_2 = \sqrt{m_t^2 - m_1^2} = \sqrt{0.757^2 - 0.4^2} = \sqrt{0.573 - 0.16} = \sqrt{0.413} = 0.643$$

# AM modulation (DSB-SC)

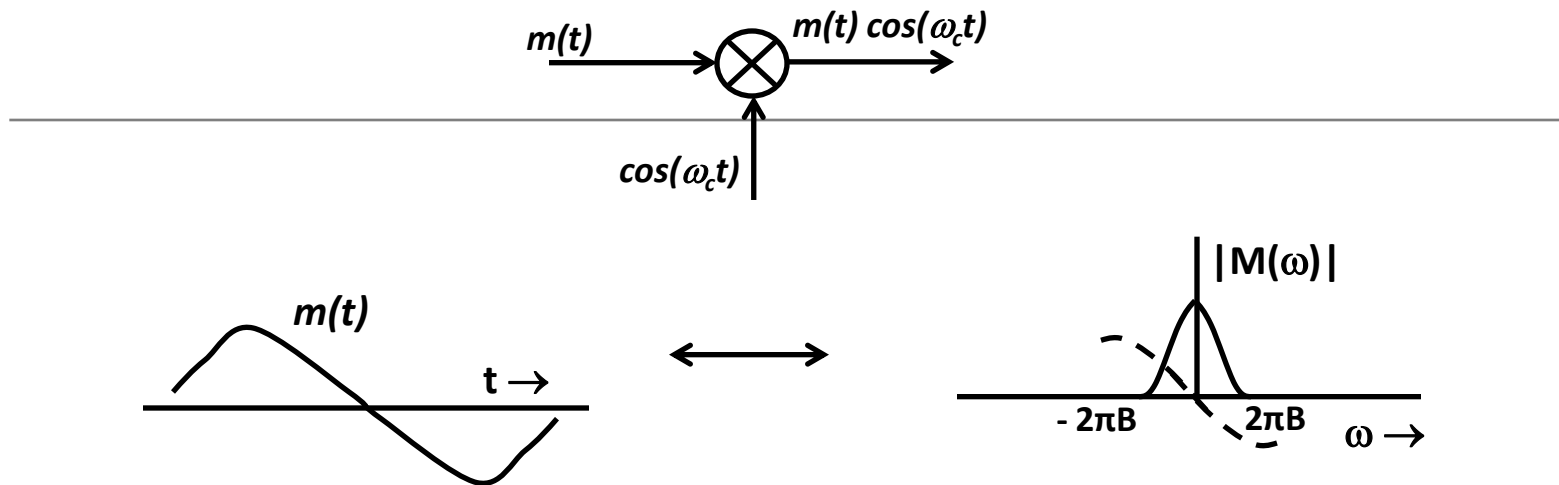
Modulation shifts the spectrum of  $m(t)$  to the carrier frequency, i.e.,

$$m(t) \leftrightarrow M(\omega)$$

$$m(t) \cos \omega_c t \leftrightarrow \frac{1}{2} [M(\omega + \omega_c) + M(\omega - \omega_c)]$$

The bandwidth of the modulated signal is  $2B$  Hz.

The modulated carrier spectrum centered at  $\omega_c$  is composed of two parts: a portion lies above  $\omega_c$ , known as USB (upper side band), and a portion below  $\omega_c$ , known as LSB (lower side band). Similarly, the spectrum centered at  $\omega_c$  has upper and lower side bands.



# AM modulation (DSB-SC)

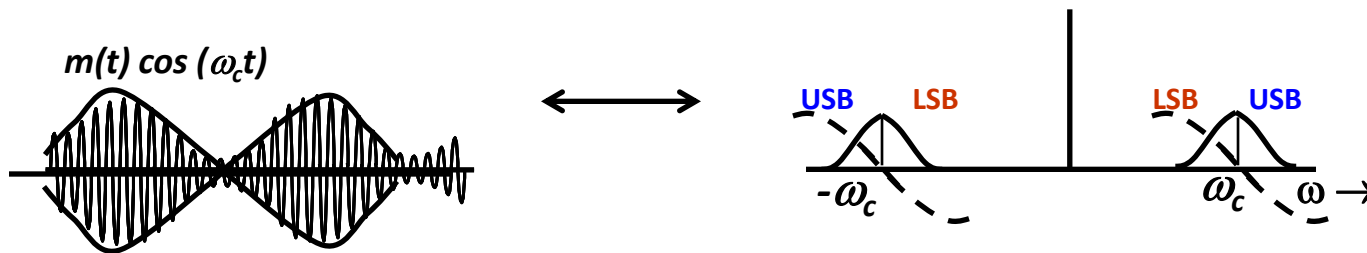
Now if  $m(t) = \cos(\omega_m t)$ , then the modulated signal

$$m(t) \cos \omega_c t = \cos \omega_m t \cos \omega_c t = \frac{1}{2} [\cos(\omega_m + \omega_c) + \cos(\omega_m - \omega_c)]$$

The component of frequency  $(\omega_c + \omega_m)$  is the upper side band, and that of frequency  $(\omega_c - \omega_m)$  is the lower sideband corresponding to the modulating signal of frequency  $\omega_m$ .

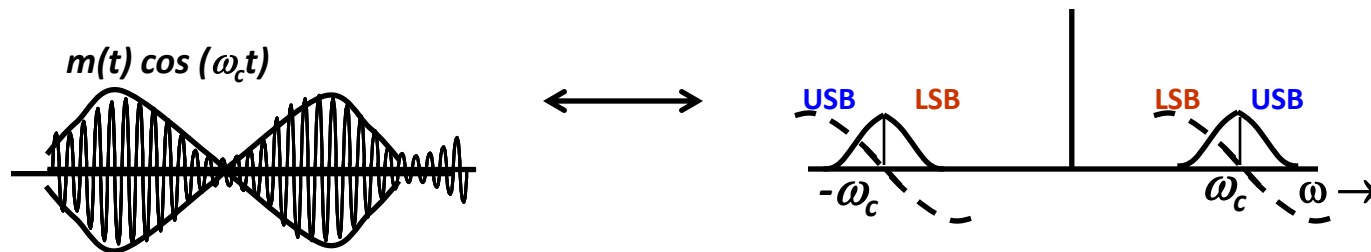
The modulated signal  $m(t) \cdot \cos(\omega_c t)$  has components of frequencies  $(\omega_c \pm \omega_m)$  but **does not have a component of the carrier frequency**.

For this reason this scheme is referred to as double-sideband suppressed-carrier (DSB-SC) modulation.

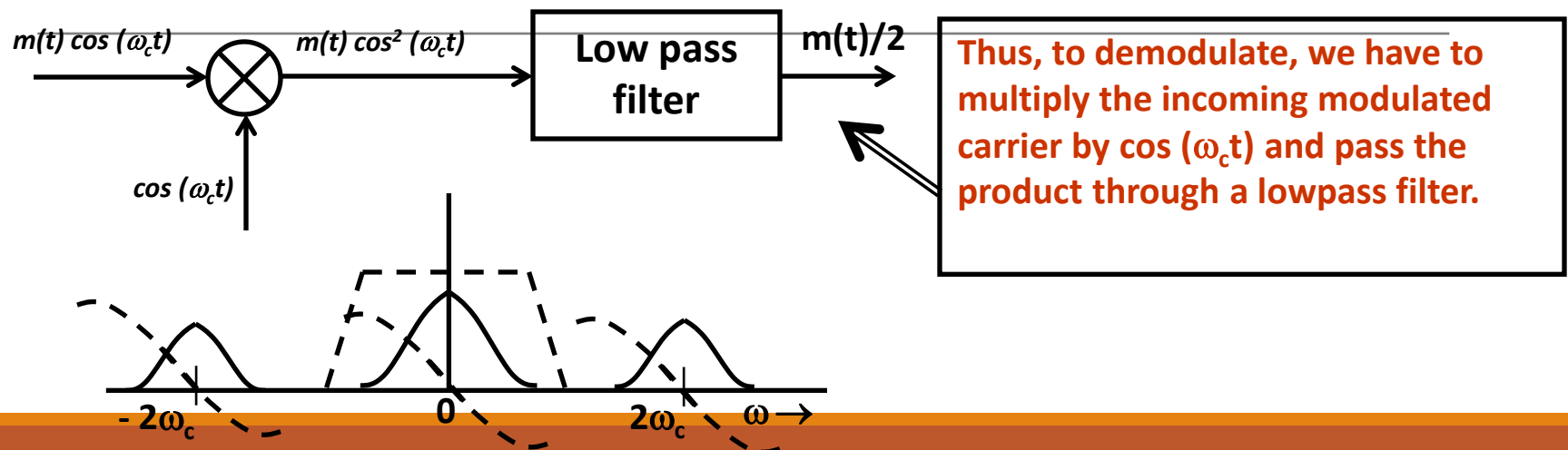


# AM modulation (DSB-SC)

The DSB-SC modulation translates the frequency spectrum. To recover the original signals, it is necessary to retranslate the spectrum to its original position. The process of retranslating the spectrum to its original position is referred to as **demodulation or detection**.



We observe that if the modulated carrier spectrum (in the above figure) is shifted again by  $\pm\omega_c$ , we get back the desired base-band spectrum plus an unwanted spectrum at  $\pm 2\omega_c$ , which can be suppressed by a lowpass filter.





# AM modulation (DSB-SC)

This conclusion can be verified directly from the following identity

$$m(t)(\cos \omega_c t)(\cos \omega_c t) = \frac{1}{2} [m(t) + m(t) \cos 2\omega_c t]$$

$$(m(t) \cos \omega_c t)(\cos \omega_c t) \leftrightarrow \frac{1}{2} M(\omega) + \frac{1}{4} [M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

A lowpass filter allows the desired spectrum  $M(\omega)$  to pass and suppress the unwanted high-frequency spectrum centered at  $\pm 2\omega_c$ .

It is interesting to note that the process at the receiver is similar to that required at the transmitter. This method of recovering the base-band signal is called **synchronous detection, or, coherent detection**, where we use a carrier of exactly the same frequency (and phase) as was used for modulation.

*Thus, for demodulation, we need to generate a local carrier at the receiver in synchronism with the carrier that was used at the modulator.*

# AM modulation (DSB-SC)

In order to avoid the overlap of  $M(\omega + \omega_c)$  and  $M(\omega - \omega_c)$ ,  $\omega_c \geq 2\pi B$ .

If  $\omega_c < 2\pi B$ , the information of  $m(t)$  is lost in the process of modulation, and it is impossible to retrieve  $m(t)$  from the modulated signal  $m(t) \cos(\omega_c t)$ .

Theoretically, the only requirement is  $\omega_c \geq 2\pi B$ . The practical factors, however, impose additional restrictions.

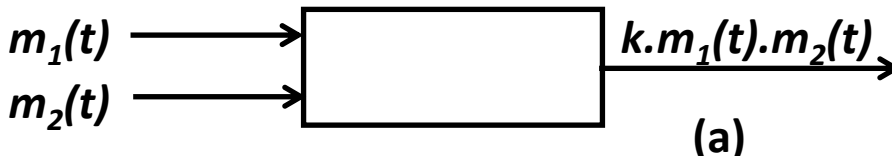
A radiating antenna can radiate only a narrowband without distortion. This means that to avoid distortion caused by the radiating antenna,  $\omega_c/2\pi B \gg 1$ .

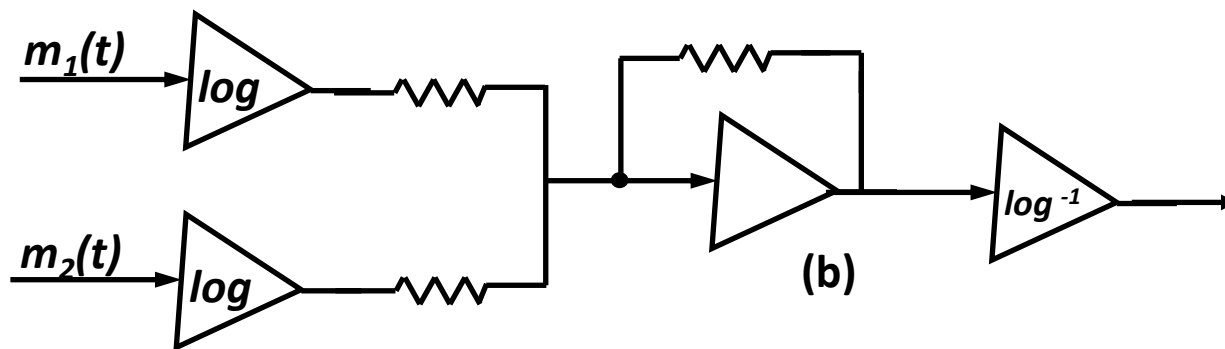
The broadcast band AM radio uses the band of 550 kHz to 1600 kHz, or a ratio of  $\omega_c/2\pi B$  roughly in the range of 100 to 300.

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# AM modulators

Modulations can be achieved in several ways. We shall restrict ourselves within some important categories of modulators.

**Multiplier modulators:**  (a)



(a) Modulation is performed directly by multiplying  $m(t)$  by  $\cos(\omega_c t)$  using an analog multiplier with O/P proportional to the product of two input signals.

For a variable gain amplifier, the gain parameter (such as  $\beta$  of a transistor) is controlled by one of the signals, say,  $m_1(t)$ . The amplifier gain is no longer constant but is  $k.m_1(t)$ .

**This type of modulator is a linear time-varying system.**

# Nonlinear modulators (DSB-SC modulator)

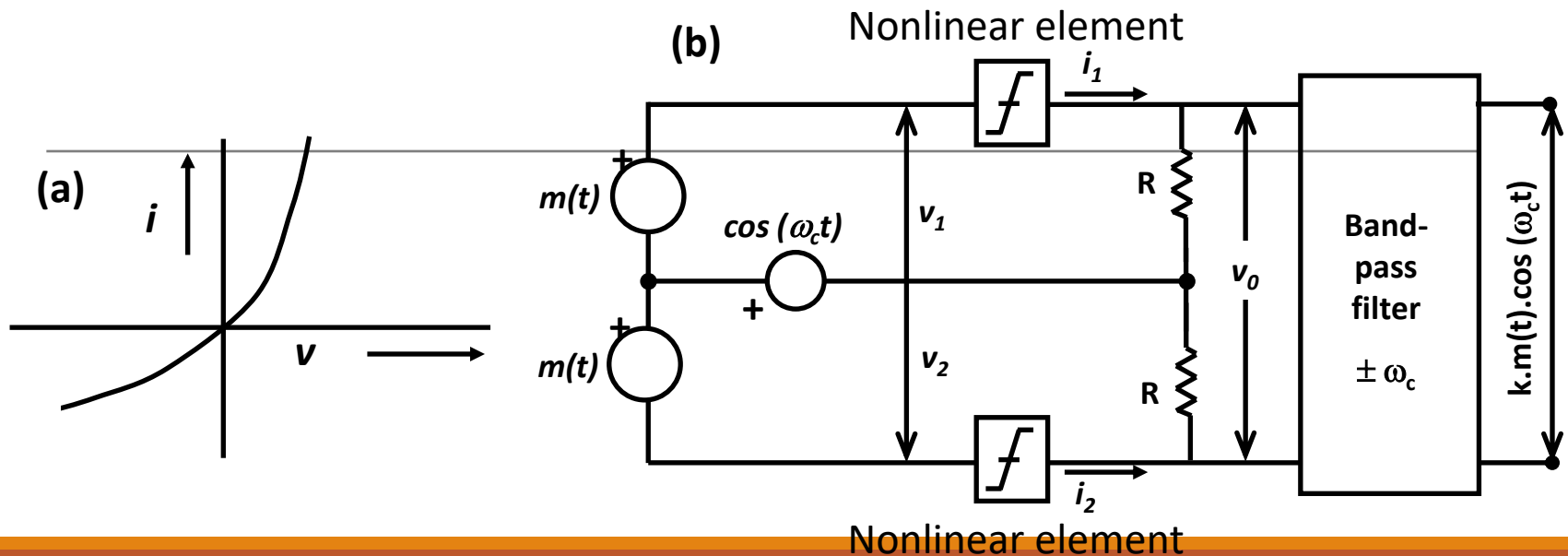
- A semiconductor diode or a transistor is a nonlinear component.

Nonlinear characteristics can be approximated by a power series as:

$$i = a.v + bv^2$$

We shall consider that the nonlinear element is in series with the resistor R as a composite element whose terminal voltage  $v$  and the current  $i$  are related by the power series as mentioned above. The voltage  $v_1$  and  $v_2$  in the following figure are given by:

$$v_1 = \cos \omega_c t + m(t) \quad \text{and} \quad v_2 = \cos \omega_c t - m(t)$$



# Modulators (nonlinear DSB-SC modulator)

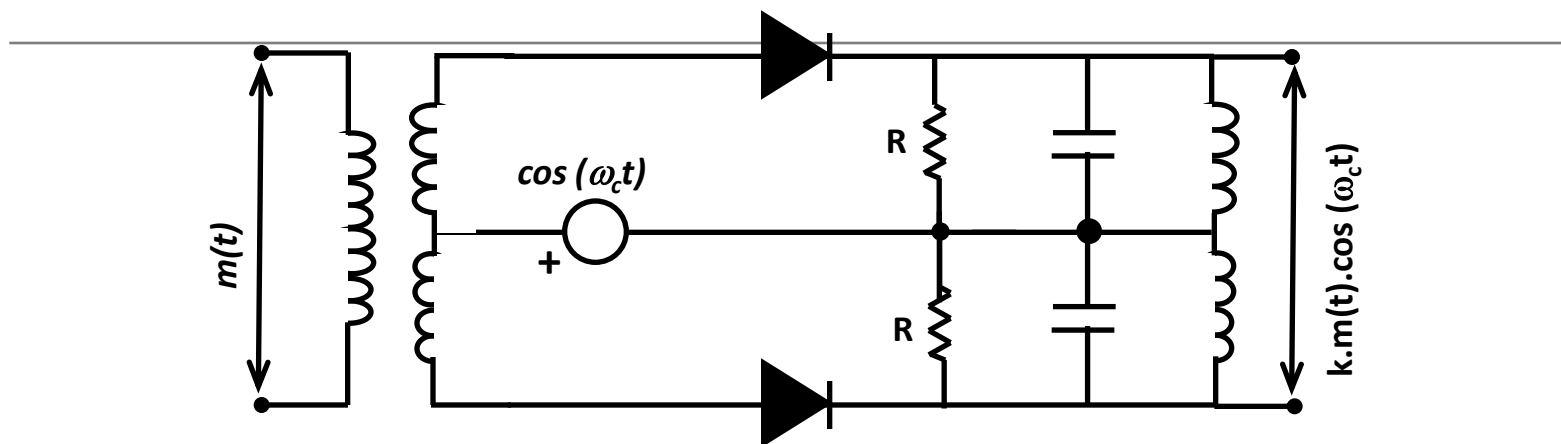
Hence, the currents  $i_1$  and  $i_2$  are given by:  $i_1 = a.v_1 + bv_1^2$

$$i_1 = a.[\cos \omega_c t + m(t)] + b[\cos \omega_c t + m(t)]^2$$

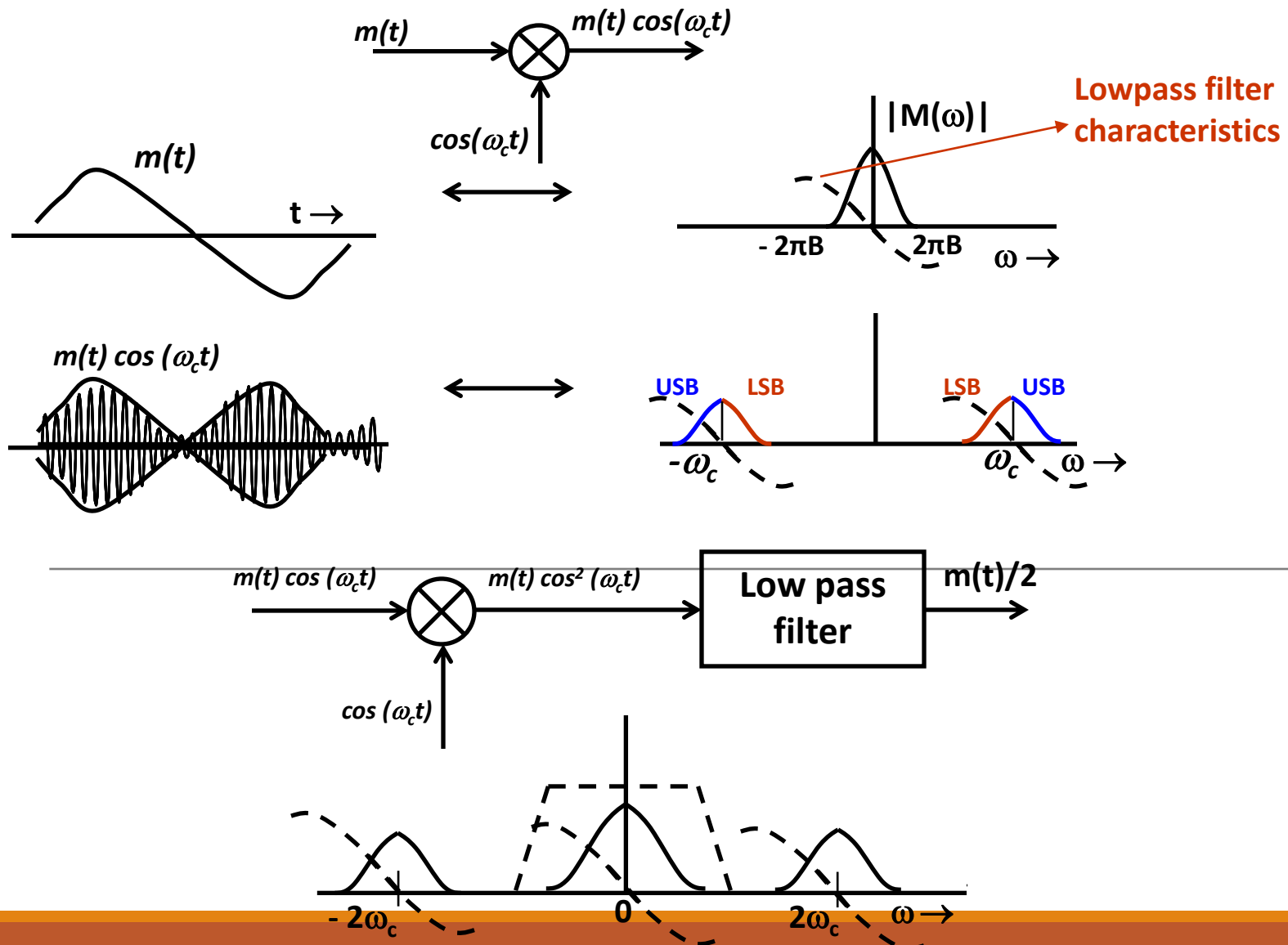
$$i_2 = a.[\cos \omega_c t - m(t)] + b[\cos \omega_c t - m(t)]^2$$

$$v_0 = i_1 R - i_2 R = 2R[2bm(t) \cos \omega_c t + am(t)]$$

The element  $a.m(t)$  in this equation can be filtered out by using a band-pass tuned to  $\omega_c$  at the output terminals. Implementation of this scheme is shown in the following figure.



# Summary: AM modulation (DSB-SC)



# Switching modulator

Multiplication operation can be replaced by simple switching operation if we realize that a modulated signal can be obtained by multiplying  $m(t)$  not only by a pure sinusoid but by any periodic signal  $\phi(t)$  of the fundamental radian frequency  $\omega_c$ .

Such a periodic function can be expressed in a Fourier series as:

$$\phi(t) = \sum_{n=0}^{\infty} C_n \cos(n\omega_c t + \theta_n) ; \text{ Hence, } m(t).\phi(t) = \sum_{n=0}^{\infty} C_n .m(t).\cos(n\omega_c t + \theta_n)$$

The spectrum of  $m(t).\phi(t)$  is the spectrum  $M(\omega)$  shifted to  $\pm \omega_c, \pm 2.\omega_c, \dots, n.\omega_c, \dots$ . If this signal is passed through a band-pass filter of bandwidth  $2B$  Hz and tuned to  $\omega_c$ , then we get the desired modulated signal  $c_1.m(t).\cos(\omega_c t + \theta_1)$ .

$$k(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n} \cos(n\omega_c t)$$

FT of pulse train with A = 1

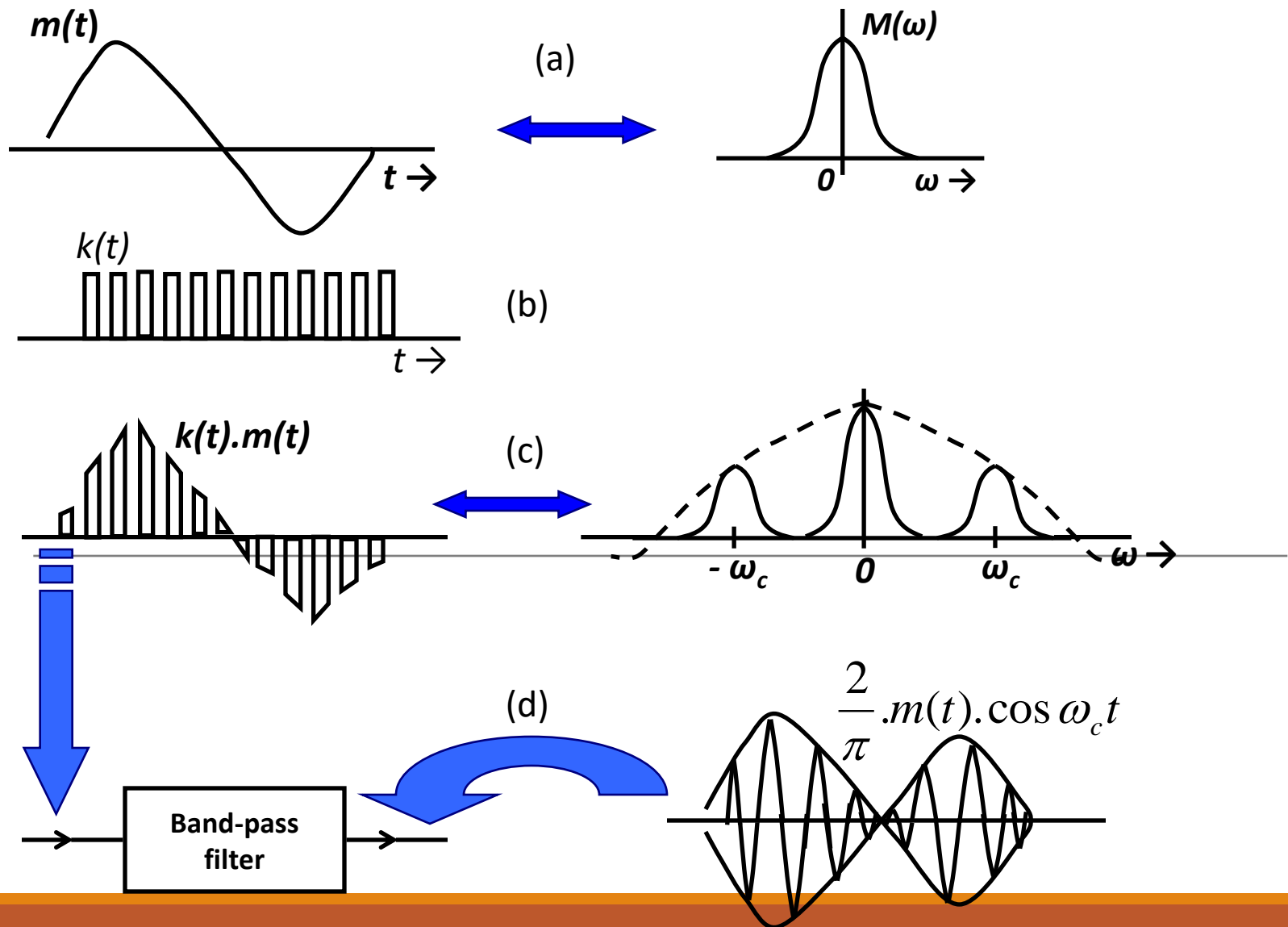
$$k(t) = \frac{1}{2} + \frac{2}{\pi} \left( \cos(\omega_c t) - \frac{1}{3} \cos(3\omega_c t) + \frac{1}{5} \cos(5\omega_c t) - \dots \right)$$

$m(t).k(t)$  is given by

$$m(t).k(t) = \frac{1}{2} m(t) + \frac{2}{\pi} \left( m(t).\cos(\omega_c t) - \frac{1}{3} .m(t).\cos(3\omega_c t) + \frac{1}{5} .m(t).\cos(5\omega_c t) - \dots \right)$$

$$m(t).k(t) \leftrightarrow \frac{1}{2} M(\omega) + \frac{1}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n} [M(\omega + n\omega_c) + M(\omega - n\omega_c)]$$

# Switching modulator for DSB-SC: graphical scheme

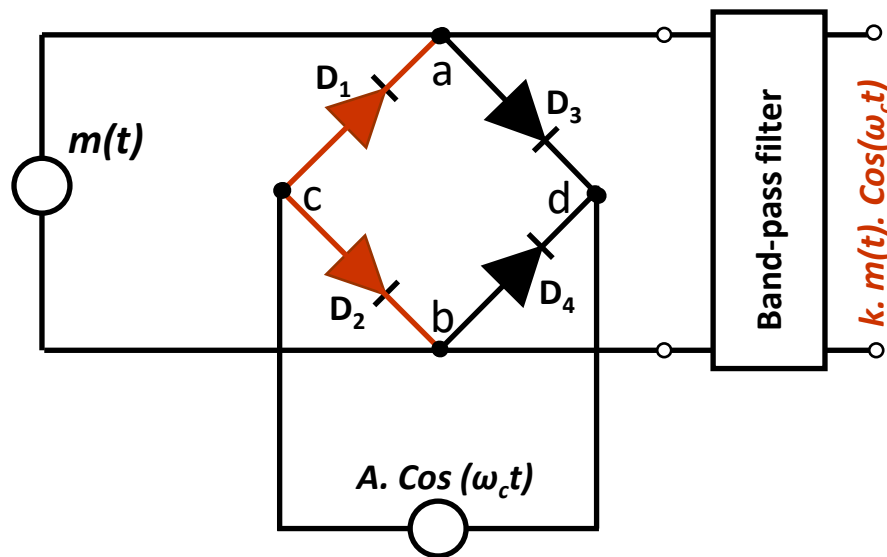




# Switching modulator

- The product  $m(t).k(t)$  and its spectrum are shown in Fig. (c).
- When signal  $m(t).k(t)$  is passed through a band-pass filter tuned a  $\omega_c$ , the O/P is the desired modulated signal
$$\frac{2}{\pi} m(t) \cos \omega_c t$$
- Multiplication of signal by a square pulse train is in reality a switching operation.
- It involves switching the signal  $m(t)$  'on' and 'off' periodically and can be accomplished by simple elements controlled by  $k(t)$ .
- This can be achieved by using a shunt-bridge diode modulator or by using a series-bridge diode modulator.

# Shunt-bridge diode modulator

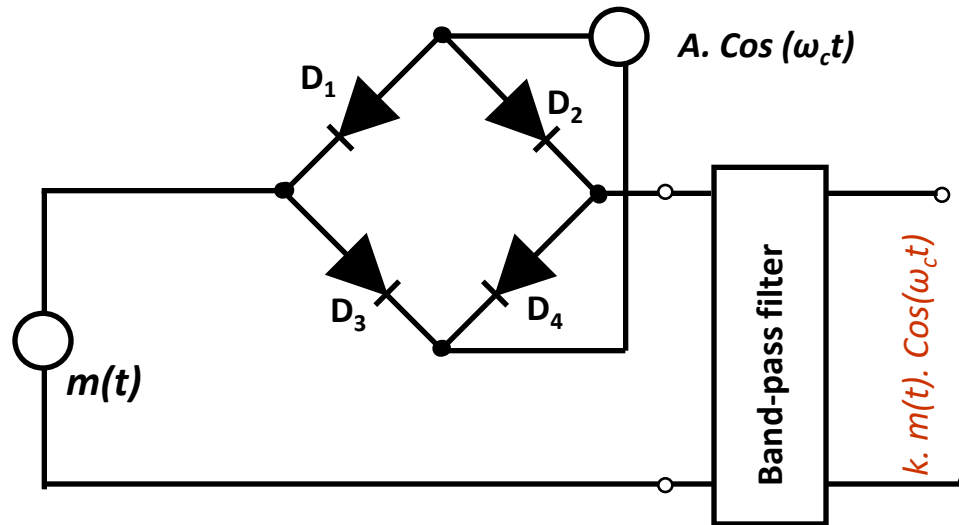


- Diodes  $D_1$ ,  $D_2$  and  $D_3$ ,  $D_4$  are matched pairs.
- The diode bridge is in parallel with  $m(t)$ , hence the name, ***shunt-bridge diode modulator***.

When the signal ( $\cos \omega_c t$ ) is of a polarity that will make the terminal 'c' positive w. r. t. d, all the diodes conduct, **assuming that the amplitude  $A \gg m(t)$** . Since  $D_1$  and  $D_2$  are matched, terminals 'a' and 'b' have the same potential, and the input to the band-pass filter is shorted during this half-cycle.

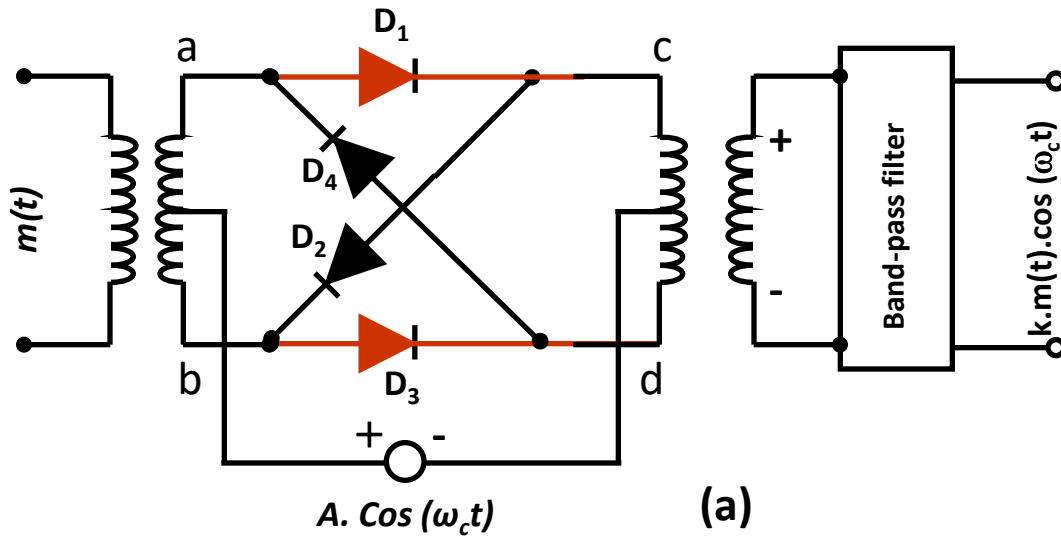
During the next half-cycle, terminal 'd' is positive with respect to 'c', and all four diodes are open, thus connecting  $m(t)$  to the input of the band-pass filter. This switching 'on' and 'off' of  $m(t)$  repeats for each cycle of the carrier. The effective input to the band-pass filter is  $m(t) \cdot k(t)$ , and O/P is the desired modulated signal  $c_1 \cdot m(t) \cdot \cos(\omega_c t)$ .

# Series-bridge diode modulator

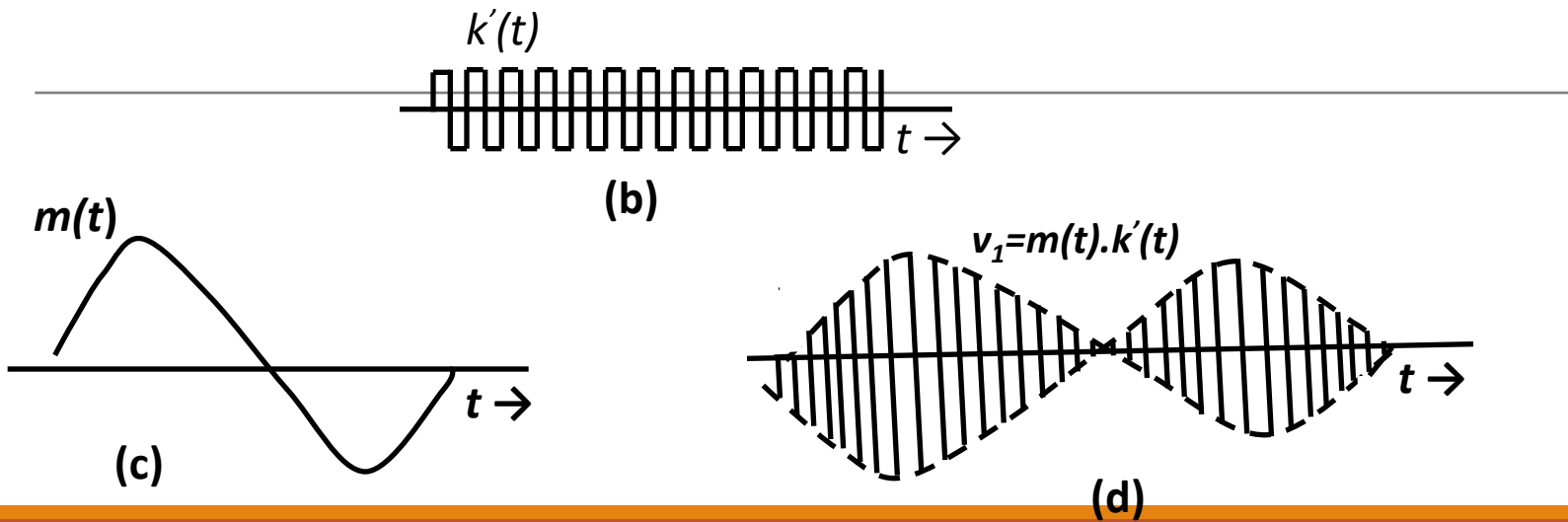


- The diode bridge is in series with  $m(t)$ , and this will accomplish the same purpose, hence the name, *series-bridge diode modulator*.

# Ring modulator



Another switching modulator is the *ring modulator* as shown here.



# Ring modulator

- The diagram of the Ring modulator is shown in the previous slide.
- During the positive half-cycles of the carrier, diodes  $D_1$  and  $D_3$  conduct, and  $D_2$  and  $D_4$  are open. Hence, terminal 'a' is connected to 'c' and 'b' is connected to 'd'.
- During the negative half-cycles of the carrier, diodes  $D_1$  and  $D_3$  are open, and  $D_2$  and  $D_4$  are shorted, thus connecting 'a' to 'd' and terminal 'b' to 'c'. Hence, the O/P is proportional to  $m(t)$  during positive half-cycle and to  $-m(t)$  during negative half-cycle.
- In effect  $m(t)$  is multiplied by a square pulse train  $k'(t)$ , shown in Fig. (b). Note that  $k'(t)$  is the same as  $k(t)$  with  $A=2$  and its dc term eliminated.

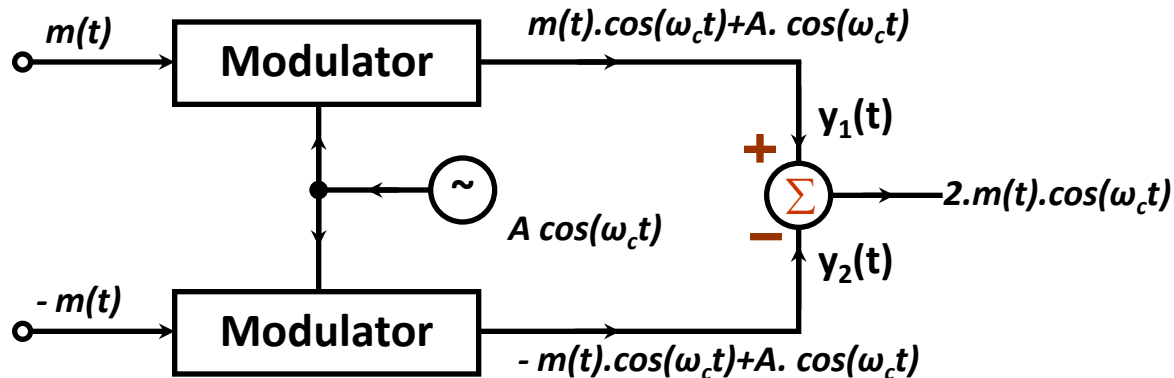
$$k'(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots} \frac{\left( (-1)^{\frac{n-1}{2}} \right)}{n} \cos(n\omega_c t) \quad m(t).k'(t) = \frac{4}{\pi} \sum_{n=1,3,5,\dots} \frac{\left( (-1)^{\frac{n-1}{2}} \right)}{n} m(t). \cos(n\omega_c t)$$

- The signal  $m(t).k'(t)$  is shown in Fig. (d). When this waveform is passed through a band-pass filter tuned to  $\omega_c$ , the filter O/P will be desired signal  $(4/\pi). m(t). \cos(\omega_c t)$

# Balanced modulator

It is easier to generate a signal of the form  $m(t).cos(\omega_c t)$ .

$m(t).cos(\omega_c t)+A. cos(\omega_c t)$  is the DSB-SC



We can generate the DSB-SC signal using two such generators in a balanced configuration that will suppress the carrier term. The O/P of the generators are:

$$y_1 = m(t) \cos \omega_c t + A \cos \omega_c t \quad y_2 = -m(t) \cos \omega_c t + A \cos \omega_c t$$

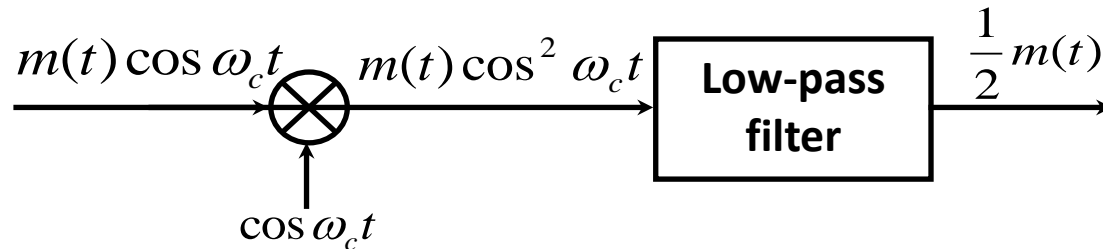
$$Y = y_1 - y_2 = 2.m(t) \cos \omega_c t$$

For perfect suppression, both modulators should be matched as closely as possible.

The nonlinear modulator with diodes can be used for this. By matching the upper and lower parts, carrier can be suppressed.

# Demodulation of DSB-SC signals

Demodulation of a DSB-SC signal is identical to modulation as it is seen from the following figure.



$$m(t)(\cos \omega_c t)(\cos \omega_c t) = \frac{1}{2} [m(t) + m(t) \cos 2\omega_c t]$$

$$(m(t) \cos \omega_c t)(\cos \omega_c t) \leftrightarrow \frac{1}{2} M(\omega) + \frac{1}{4} [M(\omega + 2\omega_c) + M(\omega - 2\omega_c)]$$

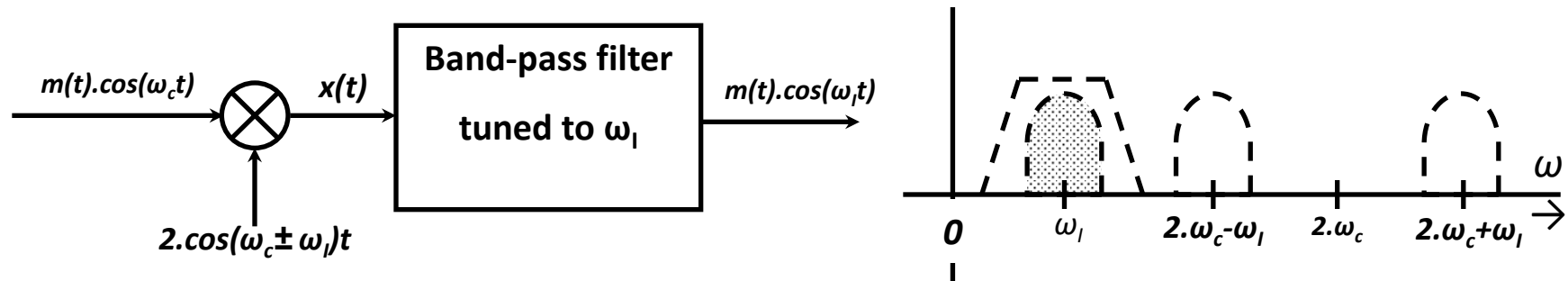
- At the receiver, we multiply the incoming signal by a local carrier of frequency and phase in synchronism with the carrier used at the modulator.
- Pass the product through a low-pass filter.
- For demodulation, receiver must generate a carrier in phase and frequency synchronous with the incoming carrier.
- These demodulators are called **synchronous, or coherent (also homodyne) demodulators**.

# Frequency mixer (or converter)

A frequency mixer, or frequency converter, is used to change carrier frequency of a modulated signal  $m(t) \cos(\omega_c t)$  from  $\omega_c$  to some other frequency  $\omega_1$ .

This can be done by multiplying  $m(t) \cos(\omega_c t)$  by  $2\cos(\omega_c \pm \omega_1)t$  and then pass the product through a band-pass filter (as shown in the following figure).

$$x(t) = 2m(t) \cos \omega_c t \cos(\omega_c \pm \omega_1)t = m(t) [\cos \omega_1 t + \cos(2\omega_c \pm \omega_1)t]$$



- The spectrum of  $m(t) \cos(2\omega_c \pm \omega_1)$  is centered at  $2\omega_c \pm \omega_1$  and is eliminated by a band-pass filter tuned to  $\omega_1$  (assuming  $\omega_1 < (\omega_c - 2\text{PIB})$ ). The filter O/P is  $m(t) \cos(\omega_1 t)$ .
- The operation of frequency mixing, or frequency conversion is **known as heterodyning**. This is identical to the operation of modulation with a modulating carrier frequency that differs from the incoming carrier frequency by  $\omega_1$ . Any one of the modulators discussed earlier can be used for frequency mixing.
- When the local carrier is  $\omega_c + \omega_1$ , the operation is called **up-conversion**, and when it is  $\omega_c - \omega_1$ , the operation is **down-conversion**.



## Difference between modulator and demodulator?

- The only difference between the modulation and demodulation is the **output filter**. In the modulator, the multiplier output is passed through a **band-pass filter** tuned to  $\omega_c$ , whereas in the demodulator, the multiplier O/P is passed through a **low-pass filter**.
- Therefore, all four types of modulators discussed earlier can also be used as demodulators, provided the band-pass filters at the O/P are replaced by low-pass filters of bandwidth B.

# Rectifier detector (demodulator)

If AM is applied to a diode and a resistor circuit, the negative part of the AM will be suppressed. The output across the resistor is a rectified version of the AM signal. In essence, the AM signal is multiplied by  $k(t)$  (the periodic pulse train). Thus, the rectified O/P  $v_R$  is:

$$v_R = [A + m(t) \cos \omega_c t] k(t)$$

$$v_R = [A + m(t)] \cos \omega_c t \left[ \frac{1}{2} + \frac{2}{\pi} \left( \cos \omega_c t - \frac{1}{3} \cos 3\omega_c t + \frac{1}{5} \cos 5\omega_c t - \dots \right) \right]$$

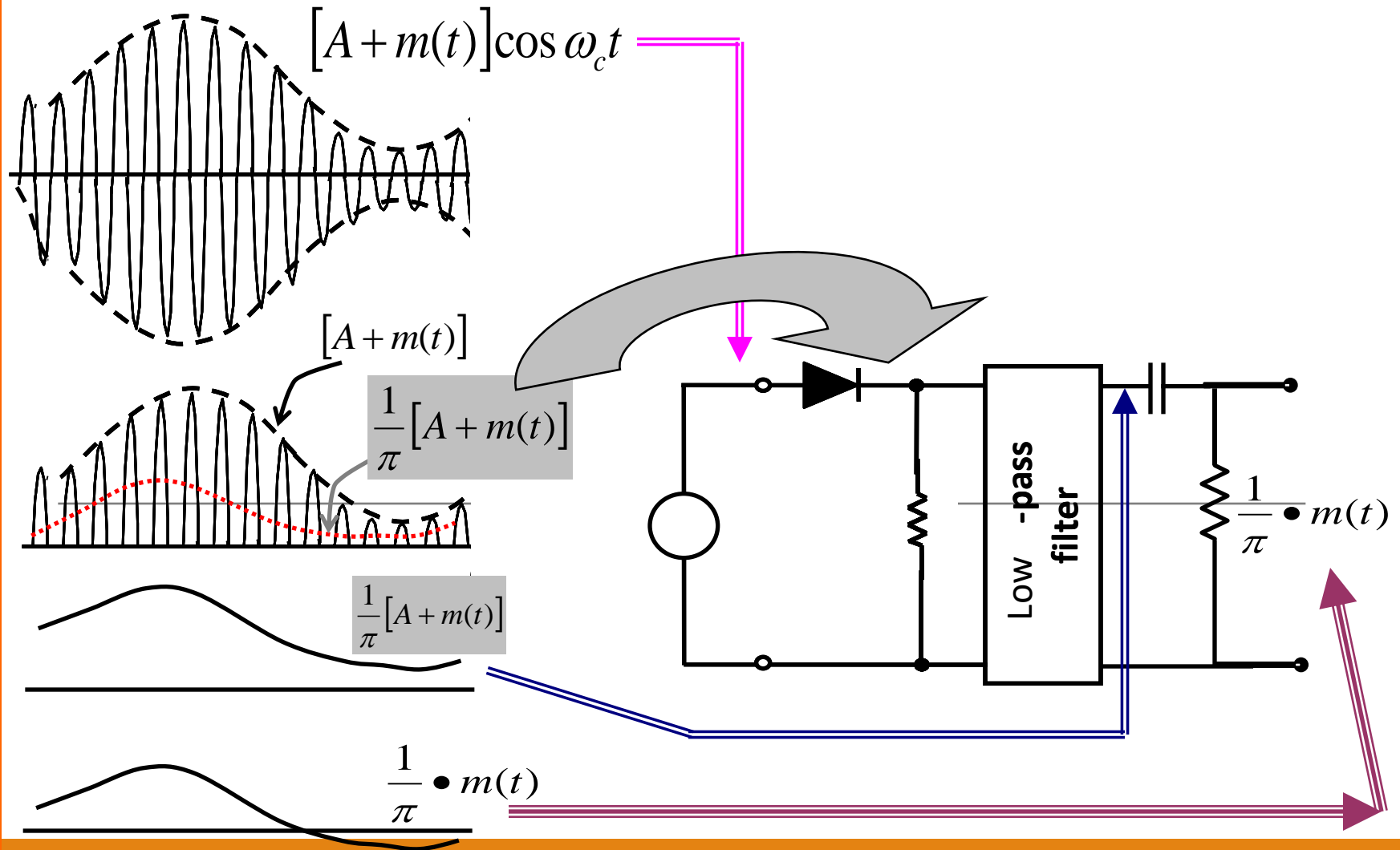
$$v_R = \frac{1}{\pi} [A + m(t)] + \text{other terms of higher frequencies}$$

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When  $v_R$  is applied to a low-pass filter of cutoff  $B$ , the O/P is  $[A+m(t)]/\pi$ , and all the other terms in  $v_R$  of frequencies higher than  $B$  are suppressed. The dc term  $A/\pi$  may be blocked by a capacitor to give the desired O/P  $m(t)/\pi$ .

The O/P can be doubled by using a full wave rectifier.

# Rectifier detector (Pictorial.....)



# Demodulator: envelope detector

In an envelope detector, the O/P of the detector follows the envelope of the modulated signal.

Functions of the envelope detector:

On the positive half cycle, the capacitor C charges up to the peak voltage of the input signal. As the peak voltage falls below this peak value, the diode is cut off, since the capacitor voltage (which is nearly the peak voltage) is greater than the input signal voltage.

The capacitor now discharges through the resistor R at a slow rate. During the next positive cycle, when the input signal becomes greater than the capacitor voltage, the diode conducts again. The capacitor again charges to the peak value of this (new) cycle. The capacitor discharges slowly during the cutoff period, thus changing the capacitor voltage very slightly.

During each positive cycle, the capacitor charges up to the peak voltage of the input signal and then decays slowly until the next positive cycle. The O/P voltage thus follows the envelope of the input. A ripple signal of frequency  $\omega_c$ , however, is caused by capacitor discharge between positive peaks. This ripple is reduced by increasing time constant RC so that the capacitor discharges very little between positive peaks ( $RC \gg 1/\omega_c$ ). Making RC too large, however, would make it impossible for the capacitor voltage to follow the envelope. Thus, RC should be large compared to  $1/\omega_c$  but should be small compared to  $1/2\pi B$ , where, B is the highest frequency in  $m(t)$ . This, incidentally, requires that  $\omega_c \gg 2\pi B$ , a condition that is necessary for a well-defined envelope.

The envelope detector O/P is  $A + m(t)$  with a ripple of frequency  $\omega_c$ . The dc term A can be blocked out by a capacitor or a simple RC high-pass filter. The ripple may be reduced further by another (low-pass) RC filter.

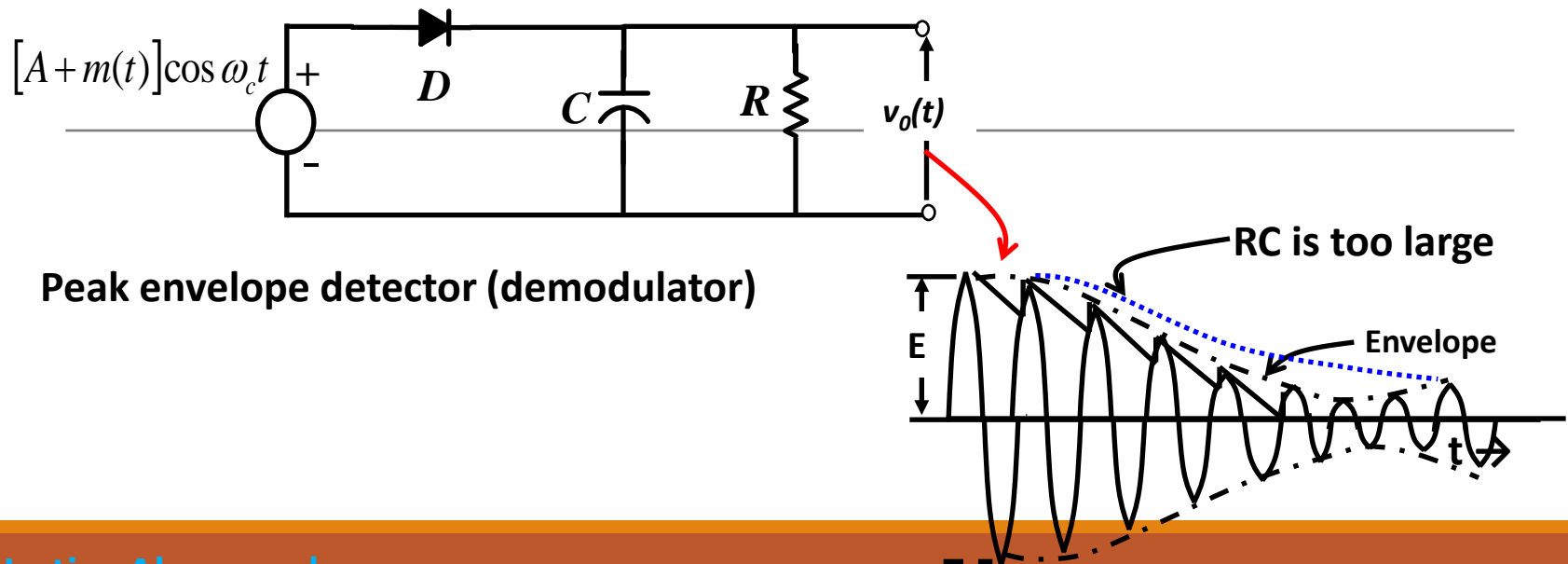
# Envelope detector

Receivers can be classified into **coherent** and **non-coherent** categories.

**Definition:** If a receiver requires knowledge of the carrier frequency and phase to extract the message signal, then it is called **coherent**.

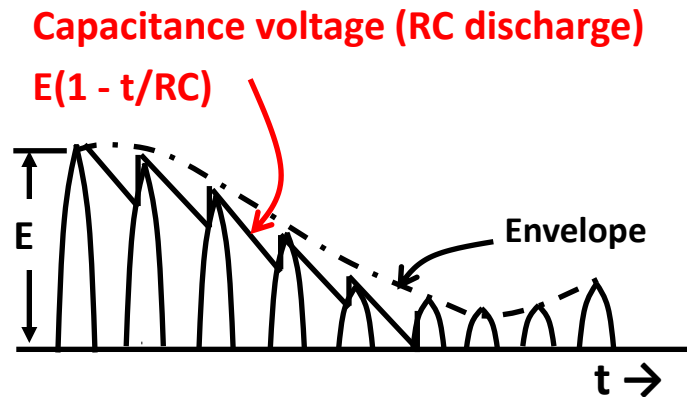
**Definition:** If a receiver does not require knowledge of the phase (only rough knowledge of the carrier frequency) to extract the message signal, then it is called **non-coherent**.

## Non-coherent demodulator (receiver) for standard AM



# Envelope detector (example...)

For tone modulation, determine the upper limit on  $RC$  to ensure that the capacitor voltage follows the envelope.



The figure shows the envelope and voltage across the capacitor.

The capacitor discharges from the peak value  $E$  at some arbitrary instant  $t=0$ . The voltage  $v_c$  across the capacitor is given by:

$$v_c = Ee^{-\frac{t}{RC}}$$

Because the time constant is much larger than the interval between the two successive cycles of the carrier ( $RC \gg 1/\omega_c$ ), the capacitor voltage  $v_c$  discharges exponentially for a short time compared to its time constant. Hence, the exponential can be approximated by a straight line obtained from the first two terms in Taylor's series of  $Ee^{-t/RC}$ .

$$v_c \cong E\left(1 - \frac{t}{RC}\right)$$

# Envelope detector (example...)

The slope of the discharge is  $-E/RC$ . In order for the capacitor to follow the envelope  $E(t)$ , the magnitude of the slope of the  $RC$  discharge must be greater than the magnitude of the slope of the envelope  $E(t)$ . Hence

$$\left| \frac{dv_c}{dt} \right| = \frac{E}{RC} \geq \left| \frac{dE}{dt} \right|$$

But the envelope  $E(t)$  of a tone-modulated carrier is

$$E(t) = A[1 + m \cos \omega_m t]$$

$$\frac{dE}{dt} = -m.A\omega_m \sin \omega_m t$$

Hence, 
$$\frac{A(1 + m \cos \omega_m t)}{RC} \geq mA\omega_m \sin \omega_m t \text{ for all } t$$

---

or, 
$$RC \leq \frac{1 + m \cos \omega_m t}{m\omega_m \sin \omega_m t} \text{ for all } t$$

The worst possible case occurs when the right-hand side is the minimum. This is found (as usual, by taking the derivative and setting it to zero) to be when  $\cos(\omega_m t) = -m$ . For this case, the right-hand side is

$$\text{Hence, } RC \leq \frac{1}{\omega_m} \left( \frac{1 - m^2}{m} \right)$$

# Square-Law detector

An AM signal can be demodulated by squaring it and then passing the squared signal through a low-pass filter. This can be seen from the fact that

$$\varphi_{AM}(t) = [A + m(t)] \cos \omega_c t \quad \text{and} \quad \varphi_{AM}^2(t) = \frac{[A^2 + 2.A.m(t) + m^2(t)]}{2} (1 + \cos 2\omega_c t)$$

The low-pass filter O/P  $y_o(t)$  is

$$y_o(t) = \frac{A^2}{2} \left[ 1 + 2 \frac{m(t)}{A} + \left( \frac{m(t)}{A} \right)^2 \right]$$

Usually,  $m(t)/A \ll 1$  for most of the time. Only when  $m(t)$  is near its peak is this violated. Hence,

---

$$y_o(t) \cong \frac{A^2}{2} + A.m(t)$$

A blocking capacitor will suppress the dc term, yielding the O/P  $A.m(t)$ . Note that the square-law detector causes signal distortion. The distortion, however, is negligible when  $m$  is small. This type of detection can be performed by any device that does not have odd symmetry. Whenever, a modulated signal passes through any nonlinearity of this type, it will create a demodulation component, whether intended or not.



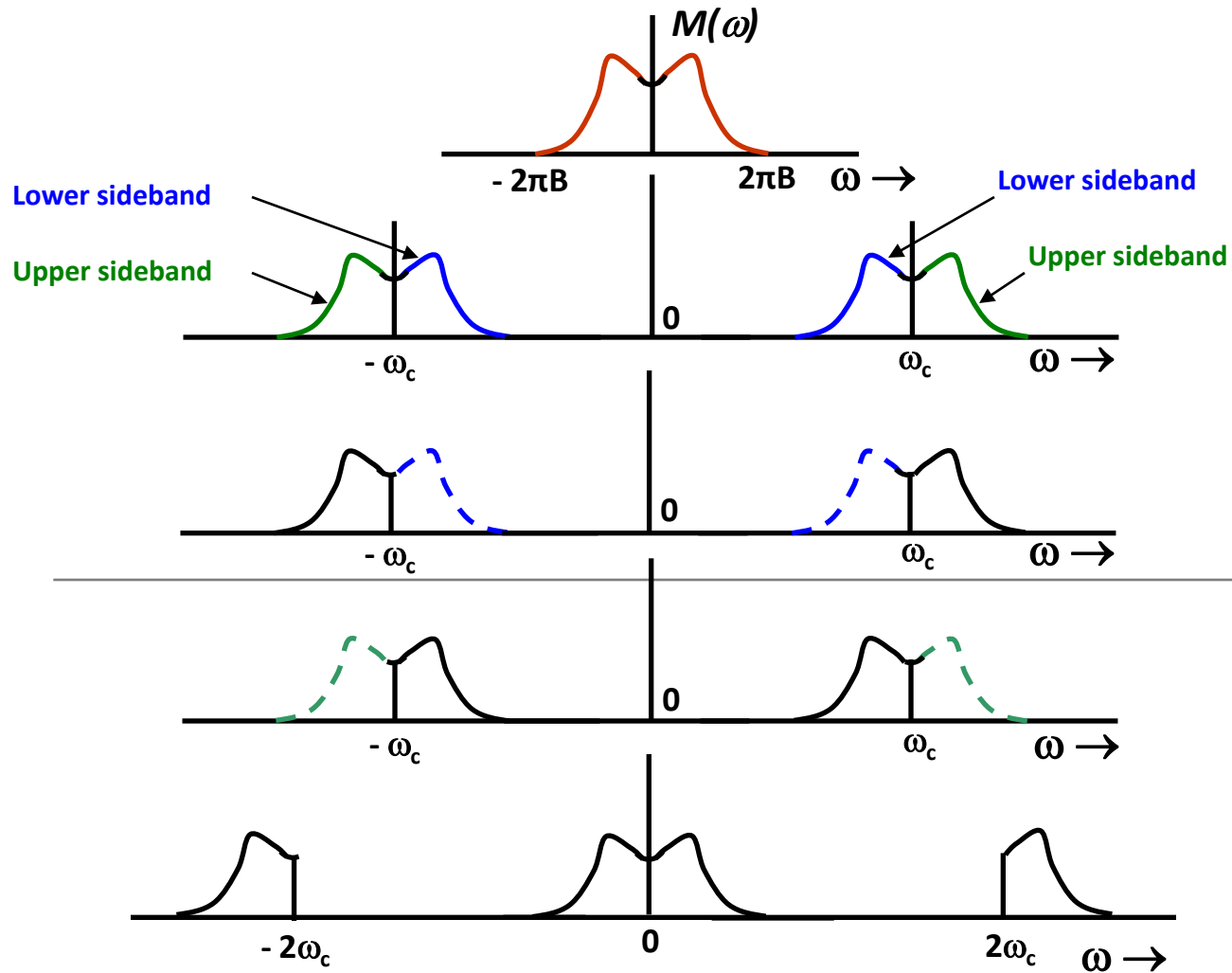
# AM: Single side band (SSB and SSB-SC)

- ❖ The DSB spectrum has two sidebands: the **upper side band** and the **lower sideband**, either of which contains the complete information of the **base-band** signal.
- ❖ A scheme in which only one sideband is transmitted is known as **single sideband (SSB)** transmission, and it requires **only half of the bandwidth** of a DSB signal.
- ❖ An SSB signal can also be **coherently (synchronously)** demodulated.
- ❖ For example, multiplication of a USB signal by  $\cos(\omega_c t)$  shifts its spectrum by  $\pm \omega_c$ . Low-pass filtering of this signal yields the desired base-band signal. The case is similar with LSB signals.
- ❖ Hence, demodulation of SSB signals is identical to that of DSB-SC signals.
- ❖ Note that we are talking about SSB signals without an additional carrier, and, hence, they are **suppressed-carrier signals (SSB-SC)**.

## Motivation:

Bandwidth occupancy can be minimized by transmitting either the lower or the upper sideband of the message signal.

# AM: Single side band (SSB)



(a) Base-band

(b) DSB

(c) LSB

(d) USB

# AM: Single side band (SSB)

To determine the time-domain expression of an SSB signal, we introduce the concept of the pre-envelope of a signal  $m(t)$ .

The spectra  $M_+(\omega) = M(\omega)u(\omega)$  and  $M_-(\omega) = M(\omega)u(-\omega)$ .

Let  $m_+(t)$  and  $m_-(t)$  be the inverse Fourier transforms of  $M_+(\omega)$  and  $M_-(\omega)$ , respectively. Because  $|M_+(\omega)|$  and  $|M_-(\omega)|$  are not even function of  $\omega$ ,  $m_+(t)$  and  $m_-(t)$  are complex signals. Also,  $|M_+(-\omega)|$  and  $|M_-(\omega)|$  are conjugates and, hence,  $m_+(-t)$  and  $m_-(t)$  are conjugates.

Also, because  $m_+(t) + m_-(t) = m(t)$

We can write,  $m_+(t) = \frac{1}{2}[m(t) + jm_h(t)]$

---

$$m_-(t) = \frac{1}{2}[m(t) - jm_h(t)]$$

To determine  $m_h(t)$ , we note that

$$M_+(\omega) = M(\omega)u(\omega) = \frac{1}{2}M(\omega)[1 + \text{sgn}(\omega)] = \frac{1}{2}M(\omega) + \frac{1}{2}M(\omega)\text{sgn}(\omega)$$

# Fourier transform of Signum function

Find the Fourier transform of the signum function  $\text{sgn}(t)$ , defined by

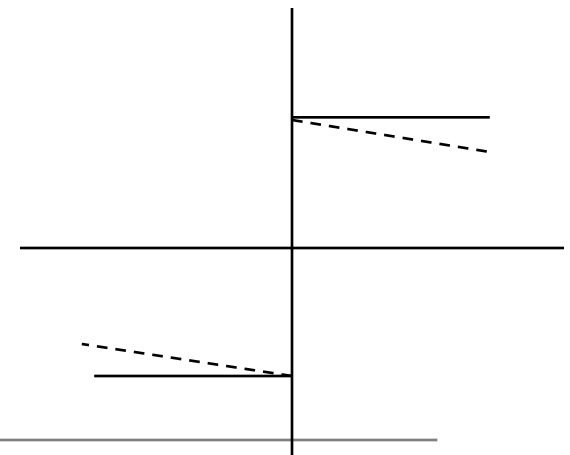
$$\text{sgn}(t) = \begin{cases} 1 & t > 0 \\ -1 & t < 0 \end{cases}$$

The transform of this function can be obtained by considering the function as a limit.

$$\text{sgn}(t) = \lim_{a \rightarrow 0} \left[ e^{-at} u(t) - e^{at} u(-t) \right]$$

$$\mathfrak{F}[\text{sgn}(t)] = \lim_{a \rightarrow 0} \left[ \int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^0 e^{at} e^{-j\omega t} dt \right]$$

$$\mathfrak{F}[\text{sgn}(t)] = \lim_{a \rightarrow 0} \left[ \int_0^{\infty} e^{-at} e^{-j\omega t} dt - \int_{-\infty}^0 e^{at} e^{-j\omega t} dt \right] = \frac{2}{j\omega}$$



Now, show that  $\frac{j}{\pi t} \leftrightarrow \text{sgn}(\omega)$

To show this, we need to use the symmetry/duality property of FT:

i.e.,  $g(t) \leftrightarrow G(\omega)$  then  $G(t) \leftrightarrow 2\pi g(-\omega)$

We already know:  $G(\omega) = \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$  and  $g(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega t} d\omega$

Thus, we can write:  $2\pi g(-t) = \int_{-\infty}^{\infty} G(x) e^{-jxt} dx \implies 2\pi g(-\omega) = \int_{-\infty}^{\infty} G(x) e^{-j\omega x} dx$

# Fourier transform of Signum function

$$\frac{2}{jt} \leftrightarrow 2\pi \operatorname{sgn}(-\omega)$$

Because  $\operatorname{sgn}(-\omega) = -\operatorname{sgn}(\omega)$ , we get

$$\frac{j}{\pi t} \leftrightarrow \operatorname{sgn}(\omega)$$

# AM: Single side band (SSB)

Comparing  $m_+(t) = \frac{1}{2} [m(t) + jm_h(t)]$  and  $M_+(\omega) = \frac{1}{2} M(\omega) + \frac{1}{2} M(\omega) \text{sgn}(\omega)$

It follows that  $jm_h(t) \leftrightarrow M(\omega) \text{sgn}(\omega)$

or,  $M_h(\omega) = -jM(\omega) \text{sgn}(\omega)$

Now,  $\frac{j}{\pi t} \leftrightarrow \text{sgn}(\omega)$

$$g(t) * f(t) = \int_{-\infty}^{\infty} g(t-\tau) f(\tau) d\tau = G(\omega) F(\omega)$$

Then from time convolution property and the relation  $\frac{j}{\pi t} \leftrightarrow \text{sgn}(\omega)$ , it follows that

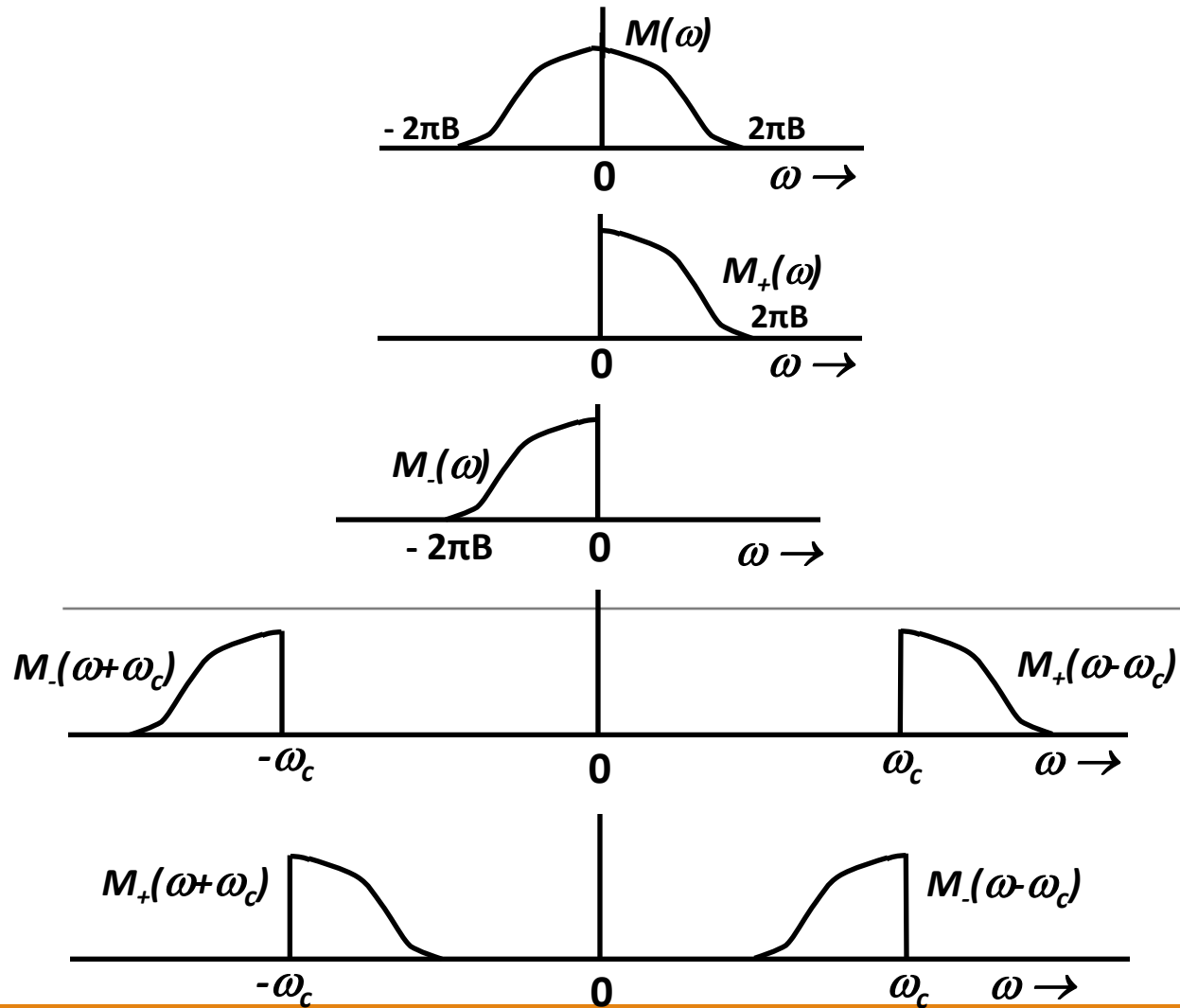
$$m_h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau$$

$$m_h(t) = \frac{1}{\pi} \left( \frac{1}{t} \right) * m(t)$$

$$m_h(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\tau)}{t-\tau} d\tau$$

The signal  $m_h(t)$  is the Hilbert transform of  $m(t)$ . From equation  $M_h(\omega) = -jM(\omega) \text{sgn}(\omega)$  it follows that if  $m(t)$  is passed through a transfer function  $H(\omega) = -j \text{sgn}(\omega)$ , then the O/P is  $m_h(t)$ , *i.e., the Hilbert transform of  $m(t)$ .*

# AM: Single side band (SSB)

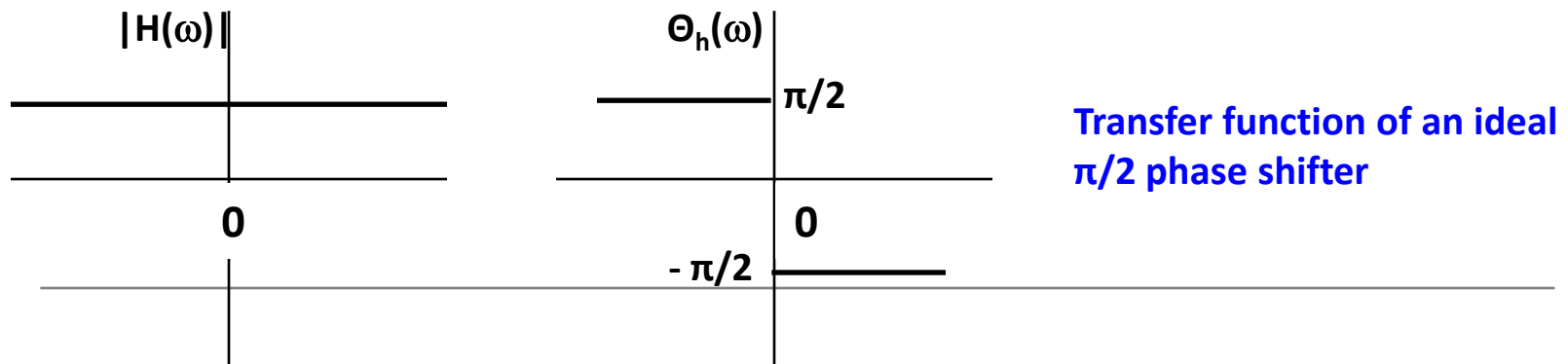


# AM: Single side band (SSB)

Because,  $H(\omega) = -j \cdot \text{sgn}(\omega)$

$$= \begin{cases} -j = 1 \cdot e^{-j\pi/2} & \omega > 0 \\ j = 1 \cdot e^{j\pi/2} & \omega < 0 \end{cases}$$

it follows that  $|H(\omega)| = 1$ , and  $\theta_h(\omega) = -\pi/2$  for  $\omega > 0$  and;  $\pi/2$  for  $\omega < 0$ , as shown in the following figure. Thus, if we delay the phase of every component of  $m(t)$  by  $\pi/2$ ,



the resulting signal is  $m_h(t)$ , the **Hilbert transform** of  $m(t)$ . A **Hilbert transformer** is just a phase shifter (that shifts each frequency component by  $-\pi/2$ ).

We can now express the SSB signal in terms of  $m(t)$  and  $m_h(t)$ . Because the USB signal  $\Phi_{USB}(\omega)$  is

$$\Phi_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

$$\Phi_{USB}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}$$



# AM: Single side band (SSB)

Substituting Eqns.  $m_+(t) = \frac{1}{2}[m(t) + jm_h(t)]$  and  $m_-(t) = \frac{1}{2}[m(t) - jm_h(t)]$

in the above equation yields  $\varphi_{USB}(t) = m(t) \cos \omega_c t - m_h(t) \sin \omega_c t$

$$\begin{aligned} & [m_+(t) + m_-(t)] \cos \omega_c t \\ & + j[m_+(t) - m_-(t)] \sin \omega_c t \end{aligned}$$

Similarly, it can be shown that

$$\varphi_{LSB}(t) = m(t) \cos \omega_c t + m_h(t) \sin \omega_c t$$

Hence, a general SSB signal  $\varphi_{SSB}(t)$  can be expressed as

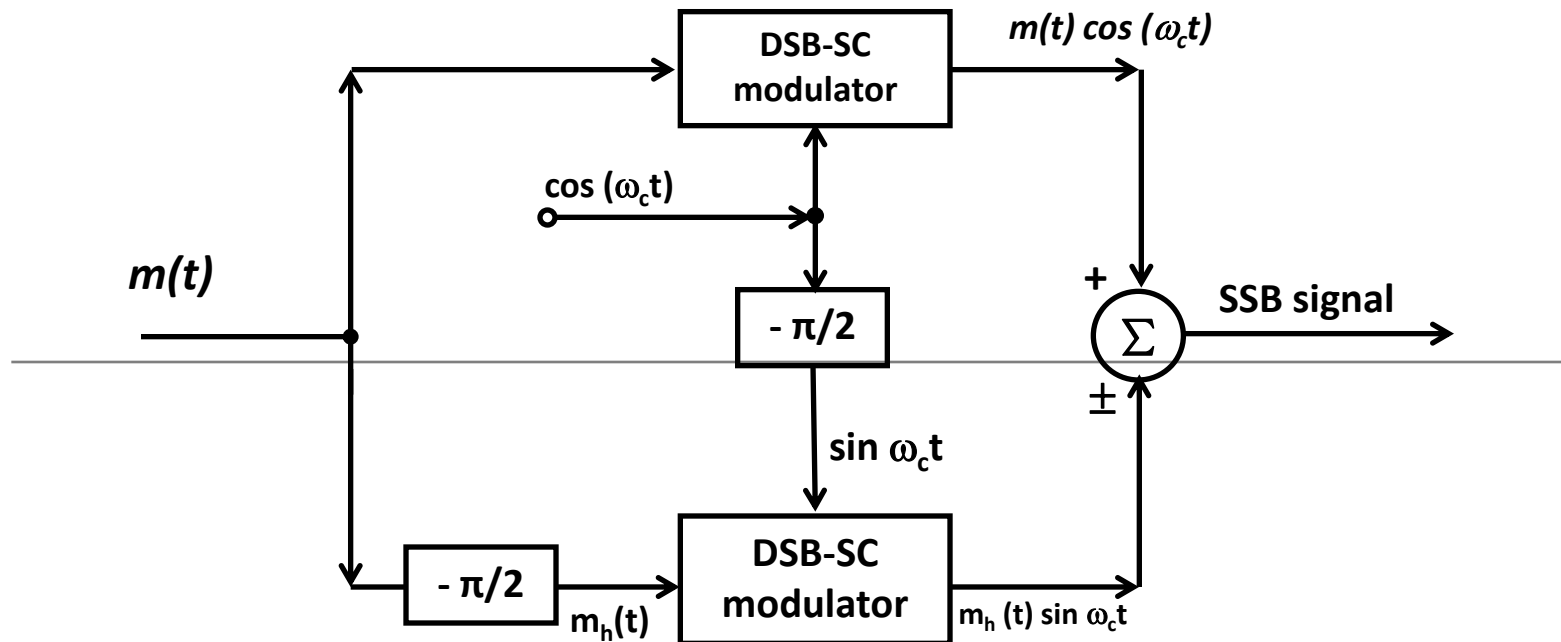
$$\varphi_{SSB}(t) = m(t) \cos \omega_c t \mp m_h(t) \sin \omega_c t$$

where the minus sign applies to the USB and the plus sign applies to the LSB.

# SSB Generator

## The phase-shift method:

SSB generator (by phase shift method) is shown in the following figure. There are  $\pi/2$  phase shifters, which delays the phase of every frequency by  $\pi/2$ . Hence, it is a Hilbert transformer. Note that an ideal phase shifter is also unrealizable. We can, at most, approximate it over a finite band.



SSB Signal: 
$$\varphi_{SSB}(t) = m(t) \cos \omega_c t \mp m_h(t) \sin \omega_c t$$

# Generation of SSB signals

Two methods are commonly used to generate SSB signals.

- method (1) uses sharp cutoff filters to eliminate the undesired sideband, and
- method (2) uses phase shifting network to achieve the same goal.
- **method (3) may also be used, provided the base-band contains little power near origin.**

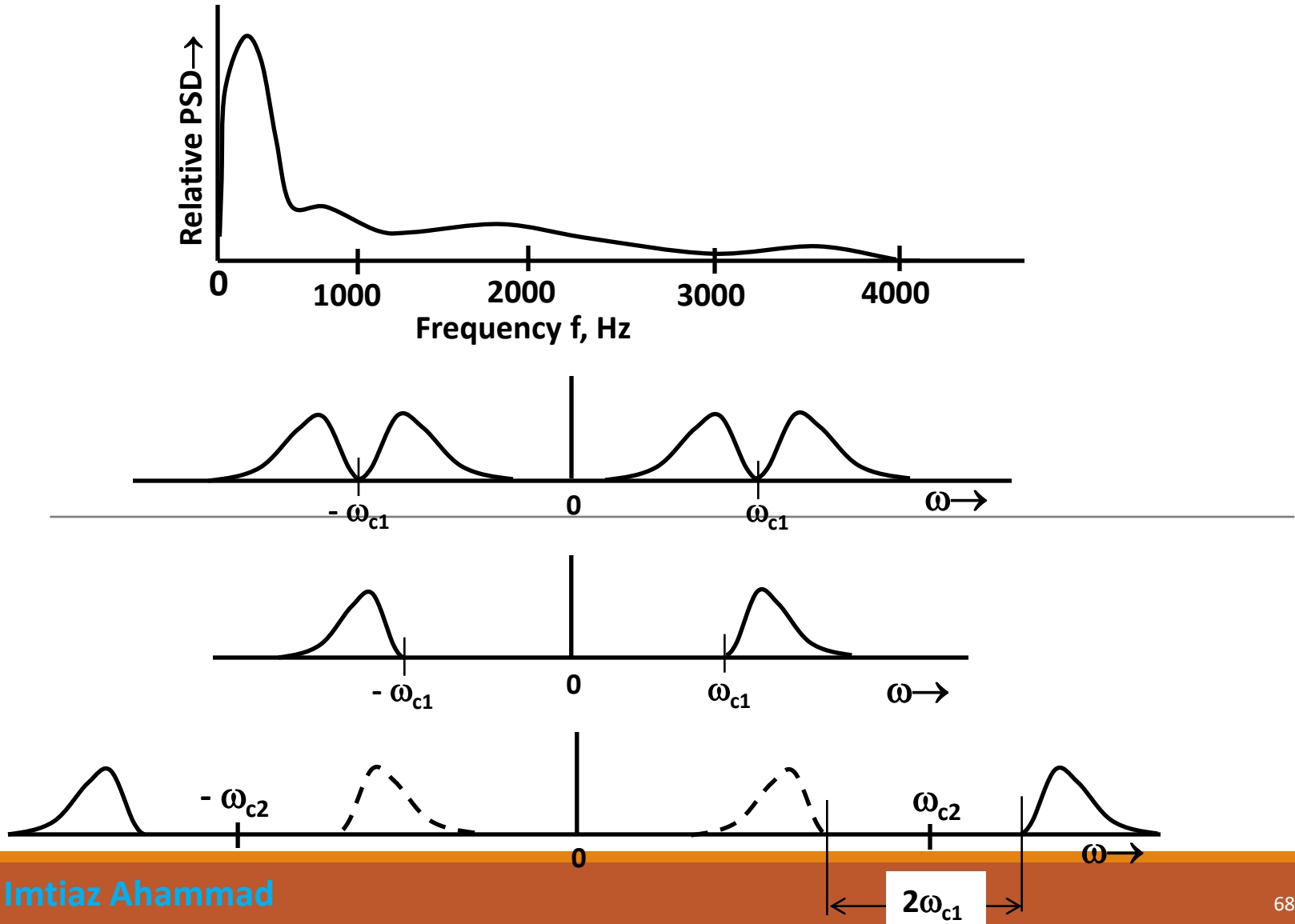
## Selective filtering:

The most commonly used method. DSB-SC signal is passed through a sharp cutoff filter to eliminate the undesired sideband.

To obtain the USB, the filter should pass all components above  $\omega_c$ , un-attenuated and completely suppress all components below  $\omega_c$ . This is impossible. There should be some separation between the passband and the stopband, Fortunately, the voice signal provides this condition, because, its spectrum shows little power content at the origin **(a)**. In addition, articulation tests have shown that for speech signals, frequency components below 300 Hz are not important affecting the intelligibility appreciably. Thus, filtering of the unwanted sideband becomes relatively easy for speech signals because we have a 600 Hz transition region around the cutoff frequency  $\omega_c$ . **(b)**

To minimize adjacent channel interference, the undesired sideband should be attenuated at least 40 dB. If the carrier frequency is too high (for example,  $f_c=10$  MHz), the ratio of the gap band (600 Hz) to the carrier frequency is small, and, thus, a transition of 40 dB in amplitude over 600 Hz could still pose a problem. In such a case, the modulation is carried out in more than one step.

# Generation of SSB signals



# Generation of SSB signals

First, the baseband signal DSB-modulates a low-frequency carrier  $\omega_{c1}$ . The unwanted sideband is easily suppressed, because the ratio of the gap band to the carrier is reasonably high. After suppression of the unwanted sideband (c), the resulting spectrum is identical to the baseband spectrum except that the gap band is now  $2\omega_{c1}$ , rather than the original 600 Hz. The first step, then amounts to increasing the gap band width to  $2\omega_{c1}$ .

In the second step, the signal (with the large gap band) DSB-modulates a carrier of high frequency  $2\omega_{c2}$ . The unwanted sideband (dotted line in (d)) is now easily suppressed because of the large gap band. If the carrier frequency is too high, the process may have to be repeated.

---

# Demodulation of SSB-SC signals

It was shown earlier that SSB-SC signals can be coherently demodulated. We can readily verify this in another way:

$$\phi_{SSB}(t) = m(t) \cos \omega_c t \mp m_h(t) \sin \omega_c t$$

Hence, 
$$\phi_{SSB}(t) \cos \omega_c t = \frac{1}{2} m(t) [1 + \cos 2\omega_c t] \mp \frac{1}{2} m_h(t) \sin 2\omega_c t$$

$$\phi_{SSB}(t) \cos \omega_c t = \frac{1}{2} m(t) + \frac{1}{2} [m(t) \cos 2\omega_c t \mp m_h(t) \sin 2\omega_c t]$$

Thus, the product  $\phi_{SSB}(t) \cos(\omega_c t)$  yields the base-band signal and another SSB signal with a carrier  $2\omega_c$ . The spectrum shows precisely this result. A low-pass filter will suppress the unwanted SSB term, giving the desired base-band signal  $m(t)/2$ . Hence, the demodulator is identical to the synchronous demodulator used for DSB-SC. Thus, any one of the synchronous DSB-SC demodulators already discussed can be used to demodulate an SSB-SC signal.

# Envelope detection of SSB signals with a carrier

We now consider SSB signals with an additional carrier (SSB+C).

$$\phi(t) = A \cos \omega_c t + [m(t) \cos \omega_c t + m_h(t) \sin \omega_c t]$$

Although  $m(t)$  can be recovered by synchronous detection (multiplying  $\phi(t)$  by  $\cos(\omega_c t)$ ) if  $A$ , the carrier amplitude, is large enough,  $m(t)$  can also be recovered from  $\phi(t)$  by envelope or rectifier detection. This can be shown by rewriting  $\phi(t)$  as

$$\phi(t) = [A + m(t)] \cos \omega_c t + m_h(t) \sin \omega_c t$$

$$\phi(t) = E(t) \cos(\omega_c t + \theta)$$

Where  $E(t)$ , the envelope of  $\phi(t)$ , is given by

$$E(t) = \sqrt{[A + m(t)]^2 + m_h^2(t)}$$

$$E(t) = A \left[ 1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2} \right]^{1/2}$$

If  $A \gg |m(t)|$ , then in general  $A \gg |m_h(t)|$ , and the terms  $m^2(t)/A^2$  and  $m_h^2(t)/A^2$  can be ignored; thus,

$$E(t) = A \left[ 1 + \frac{2m(t)}{A} \right]^{1/2}$$

Using binomial expansion and discarding higher-order terms ( $m(t)/A \ll 1$ ), we get

$$E(t) = A \left[ 1 + \frac{m(t)}{A} \right] \Rightarrow E(t) = A + m(t)$$

# Demodulation of SSB-SC signals

It is evident that for a large carrier, the envelope of  $\phi(t)$  has the form of  $m(t)$ , and the signal can be demodulated by an envelope detector.

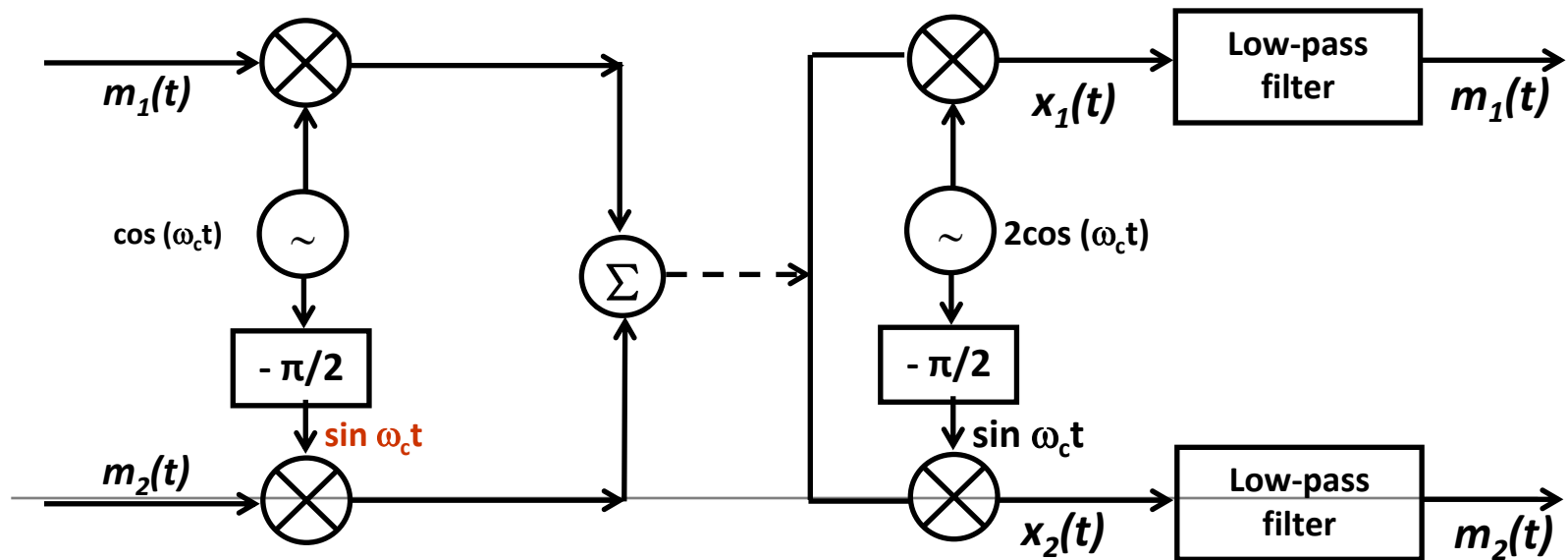
In AM, envelope detection requires the condition  $A \geq -m(t)_{\min}$ , whereas for SSB + C, the condition is  $A \gg |m(t)|$ .

Hence, in SSB, the required carrier amplitude is much larger than that is AM, and, consequently, the efficiency of SSB + C is much less than that of AM.



# Quadrature amplitude modulation (QAM)

DSB signals occupy twice the bandwidth required for SSB signals. This disadvantage can be overcome by transmitting **two DSB signals** using carriers of same frequency but in phase quadrature. Both modulated signals occupy the same band. The two base-band signals can be separated at the receiver by synchronous detection using two local carriers in phase quadrature. This can be shown by considering the multiplier O/P  $x_1(t)$  of channel 1 as the following:



$$x_1(t) = 2[m_1(t) \cos \omega_c t + m_2(t) \sin \omega_c t] \cos \omega_c t$$

$$x_1(t) = m_1(t) + m_1(t) \cos 2\omega_c t + m_2(t) \sin 2\omega_c t$$

The last two terms are suppressed by the low-pass filter, yielding the desired O/P  $m_1(t)$ . Similarly, the O/P of channel 2 can be shown to be  $m_2(t)$ . This scheme is known as quadrature amplitude modulation (QAM), or quadrature multiplexing.

# Quadrature amplitude modulation (QAM)

Thus, two signals can be transmitted over a bandwidth of  $2B$  either by using *SSB transmission and frequency-division multiplexing* or by using *DSB transmission and quadrature multiplexing*.

## Why QAM multiplexing is not preferable?

- A slight error in the phase of the carriers at the demodulator in QAM will not only result in loss of signal but will also lead to interference between the two channels.
- Similar difficulties arise when the local frequency is in error.
- In addition, unequal attenuation of the upper and lower sidebands during transmission also leads to crosstalk or co-channel interference. Unequal attenuation of the channel will destroy the symmetry of the magnitude spectrum about  $\pm \omega_c$ , and the receiver signal component  $\varphi_1(t)$  from  $m_1(t)$  will have a form as:

$$\varphi_1(t) = m_{c1}(t) \cos \omega_c t + m_{s1}(t) \sin \omega_c t$$

instead of  $m_1 \cos(\omega_c t)$ .

# Quadrature amplitude modulation (QAM)

- Demodulation of this signal will result in  $m_{c1}(t)$  in channel 1 and  $m_{s1}(t)$  in channel 2.
  - Both  $m_{c1}(t)$  and  $m_{s1}(t)$  are derived from  $m(t)$ .
  - Hence, channel 1 will receive the desired signal in a distorted form, and channel 2 will receive an interference signal.
  - Similarly, the component  $\phi_2(t)$  from  $m_2(t)$  will yield the distorted version of desired signal in channel 2 and an interference signal in channel 1.
-

# Quadrature amplitude modulation (QAM)

- **Why SSB multiplexing is preferable?**

If we were to multiplex two SSB signals on adjacent channels, a phase error in the carriers would cause distortion in each channel but not interference. Similarly, the channel non-idealities would cause distortion in each channel without causing interference. The only interference in SSB multiplexing is caused by incomplete suppression of the unwanted sideband.

**For this reason, SSB multiplexing is preferred to quadrature multiplexing.**

## Typical use QAM:

- Quadrature multiplexing is used in color television to multiplex the so-called chrominance signals, which carry the information about colors. There, the synchronization pulses are transmitted to keep the local oscillator at the right frequency and phase.
- QAM is also used in digital signal transmission.

# Quadrature amplitude modulation (QAM)

To transmit base-band pulses at a rate of 2400 bits/second, we need a minimum bandwidth of 1200 Hz. In practice, however, using Nyquist's first criterion pulses, the required bandwidth is  $(1+r) \cdot 1200$ , where  $r$  is a roll-off factor. In this particular application, 12.5% roll-off ( $r=0.125$ ) is used. This gives a bandwidth of 1350 Hz. Modulation doubles the required bandwidth to 2700 Hz. Thus, we need a bandwidth of 2700 Hz to transmit data at a rate of 2400 bits/second. Use of quadrature multiplexing will double the rate over the same bandwidth. Note that in the above method, we are transmitting 2 PSK signals using carriers in phase quadrature. For this reason, it is also known as quadrature PSK (QPSK).

We can increase the transmission rate further by using M-ary QAM. One practical case with  $M=16$  uses the following 16 pulses (16 symbols).

$$p_i(t) = a_i p(t) \cos \omega_c t + b_i p(t) \sin \omega_c t$$

$$p_i(t) = r_i p(t) \cos(\omega_c t + \theta) \quad i=1, 2, \dots, 16$$

where  $p(t)$  is properly shaped base-band pulse.

Since  $M=16$ , each pulse can transmit the information of  $\log_2 16=4$  binary digits. This can be done as follows: there are 16 possible sequences of four binary digits and there are 16 combinations  $(a_i, b_i)$  or  $(r_i, \theta_i)$ . Therefore, one single pulse  $r_i p(t) \cos(\omega_c t + \theta_i)$  transmits four bits. Then a normal transmission line that can transmit 2400 bits/second can now transmit 9600 bits/second.

# Quadrature amplitude modulation (QAM)

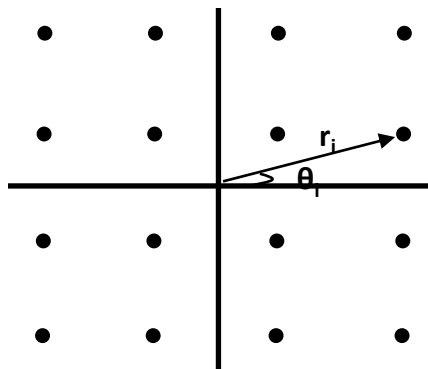
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Modulation as well as demodulation can now be performed by using the system as shown in the figure (a). The inputs are  $m_1(t)=a_i \cdot p(t)$  and  $m_2(t)=b_i \cdot p(t)$ . The two O/P at the demodulator are  $a_i \cdot p(t)$  and  $b_i \cdot p(t)$ . From the knowledge of  $(a_i, b_i)$ , we can determine the four transmitted bits.

At each end of the telephone line, we need a modulator and demodulator to transmit as well as to receive data. The two devices, modulator and demodulator are usually packaged in one unit called modem.

# Comparison of various AM systems

So far, we have discussed various aspects of AM and AM-SC (DSB-SC and SSB-SC) systems.

- AM system has advantage over DSB-SC and SSB-SC systems.
- The detectors required for AM is relatively simpler (rectifier or envelope detector) than the suppressed-carrier systems.
- AM signals are easier to generate at higher power levels, as compared to suppressed-carrier signals.
- The balanced modulators required in the latter are difficult to design.
- Suppressed-carrier systems have an advantage over AM in that they require less power to transmit the same information.
- Under normal conditions, the carrier takes up to  $\geq 75\%$  of the total transmitted power, requiring an expensive transmitter for AM.
- For suppressed-carrier, the receiver is more complex and expensive.
- For point-to-point communication, where there are only few receivers for one transmitter, the complexity in a receiver is justified, whereas for public broadcast systems, where there are millions of receivers for each transmitter, AM is obvious choice.

# Comparative study of various AM systems

- AM signals suffers from the phenomenon of fading. Fading is strongly frequency dependent, i.e., various frequency components suffer different attenuation and non-linear phase-shifts. This is known as **selective fading**.
- The effect of fading is more serious on AM signals than on AM-SC signals, because in AM the carrier must maintain a certain strength in relation to the sidebands.
- The effect of selective fading becomes pronounced at higher frequencies. Therefore, suppressed-carrier systems are preferred at higher frequencies.
- The AM system is generally used for medium-frequency broadcast transmission.

## DSB-SC and SSB-SC:

- **SSB-SC needs only the half of the bandwidth needed for DSB-SC. Although this difference can be balanced out by quadrature multiplexing two DSB-SC signals, practical difficulties of crosstalk are much serious in quadrature multiplexing.**
- **Frequency and phase errors in the local carrier used for demodulation have more serious effects in DSB-SC than in SSB-SC, particularly for voice signals.**
- **Selective fading disturbs the relationship of the sidebands in DSB-SC and causes more serious distortion than in the case of SSB-SC, where only one sideband exists.**

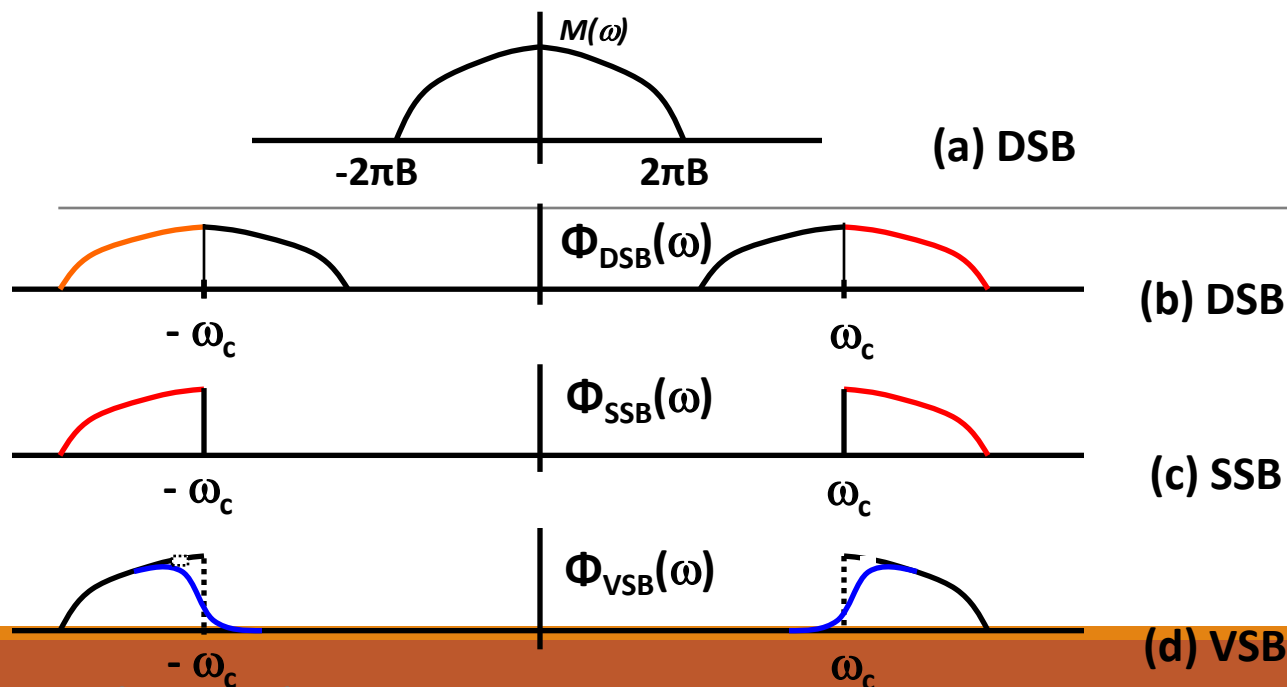
**DSB-SC is rarely used in audio communication. Long-haul telephone systems use SSB-SC multiplexed systems with a pilot carrier. In short-haul communication, DSB is sometime used. PCM replaces both.**



# AM: vestigial sideband (VSB)

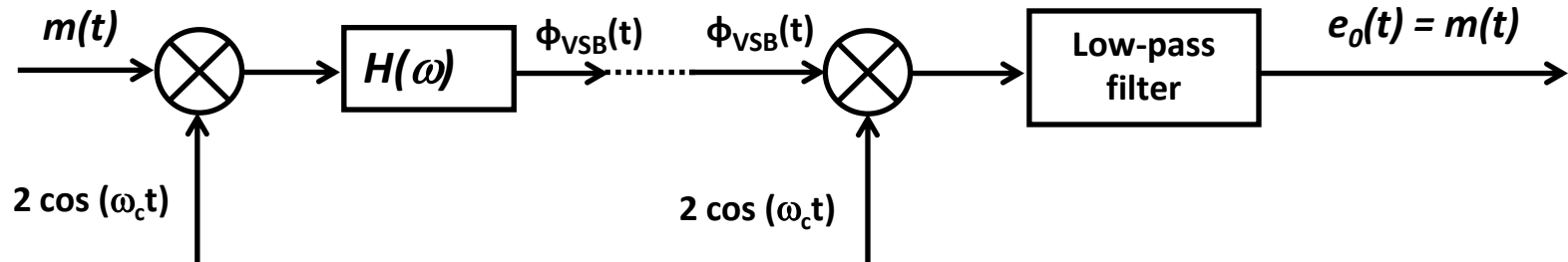
A vestigial-sideband system is a compromise between DSB and SSB. It inherits the advantages of DSB and SSB but avoids their disadvantages. VSB signals are relatively easy to generate, and, at the same time, their bandwidth is only slightly (typically 25%) greater than that of SSB signals.

In VSB, instead of rejecting one sideband completely (as in SSB), a gradual cutoff of one sideband is adopted (as shown in the figure (d)). The roll-off characteristic of the filter is such that the partial suppression of the transmitted sideband (the upper sideband (d)) in the neighborhood of the carrier is exactly compensated for by the partial transmission of the corresponding part of the suppressed sideband (the lower sideband in (d)).



# VSB modulator and demodulator

Because of the spectral shaping, the base-band signal can be recovered exactly by a synchronous detector (as shown in the following figure). If a large carrier is transmitted along with the VSB signal, the base-band signal can be recovered by an envelope (or a rectifier) detector.



**Transmitter**

**Receiver**

To determine  $H(\omega)$ , the vestigial shaping filter, to produce VSB from DSB, we have

$$\phi_{VSB}(\omega) = [M(\omega + \omega_c) + M(\omega - \omega_c)]H(\omega)$$

We require that  $m(t)$  be recoverable from  $\phi_{VSB}(t)$  using synchronous demodulation of the latter. This is done by multiplying the incoming VSB signal  $\phi_{VSB}(t)$  by  $2 \cdot \cos(\omega_c t)$ . The product  $e_o$  is given by

$$e_o(t) = 2\phi_{VSB}(t) \cos \omega_c t \leftrightarrow [\phi_{VSB}(\omega + \omega_c) + \phi_{VSB}(\omega - \omega_c)]$$

Substitution of  $\phi_{VSB}$  in the above equation and eliminating the spectra at  $\pm 2\omega_c$  (suppressed by a low-pass filter), the output  $e_o(t)$  at the low-pass filter (figure above) is given by

$$e_o(t) \leftrightarrow M(\omega)[H(\omega + \omega_c) + H(\omega - \omega_c)]$$

# VSB filter

For distortion-less reception, we must have

$$e_0(t) \leftrightarrow CM(\omega) \quad C \text{ is a constant}$$

Choosing  $C = 1$ , we have  $[H(\omega + \omega_c) + H(\omega - \omega_c)] = 1 \quad |\omega| \leq 2\pi B$

For any real filter,  $H(-\omega) = H^*(\omega)$ .

$$[H(\omega_c + \omega) + H^*(\omega_c - \omega)] = 1 \quad |\omega| \leq 2\pi B$$

$$[H(\omega_c + x) + H^*(\omega_c - x)] = 1 \quad |x| \leq 2\pi B$$

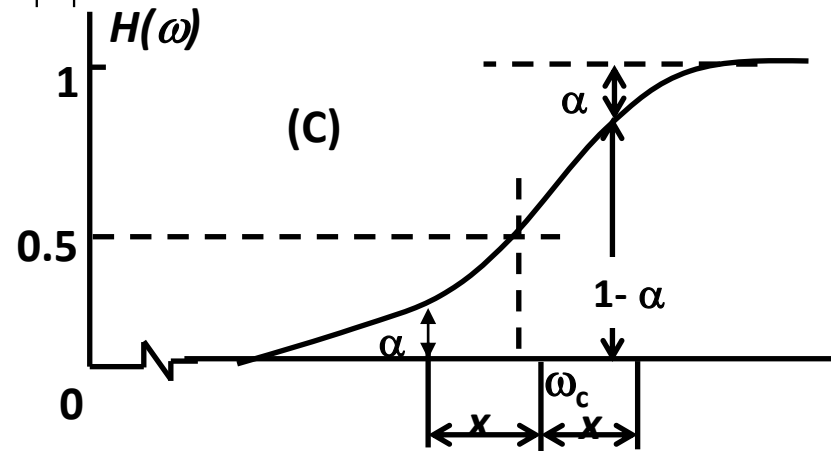
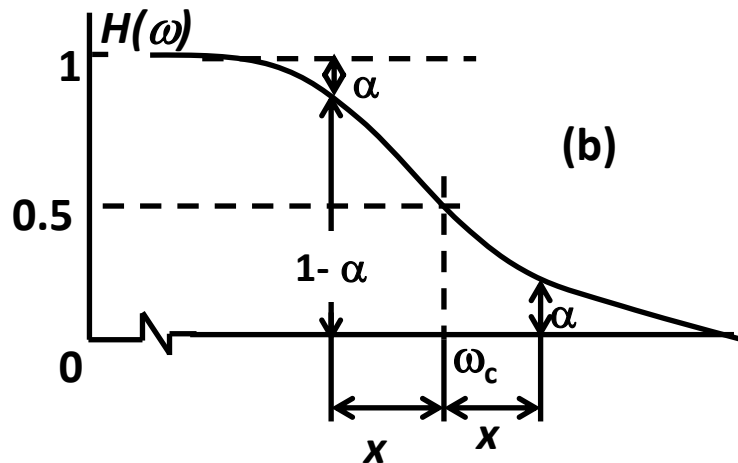
This is precisely the vestigial filter.

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# AM: VSB filter

If we consider a filter with a transfer function of the form  $|H(\omega)|e^{-j\omega t}$  the term  $e^{-j\omega t}$  represents a pure delay. Hence, only  $|H(\omega)|$  needs to satisfy the above equation. Because  $|H(\omega)|$  is real, the above equation implies that

$$|H(\omega_c + x)| + |H(\omega_c - x)| = 1 \quad |x| \leq 2\pi B$$



VSB filter characteristic LSB

VSB filter characteristic USB

Figures (b) and (c) show two possible forms of  $|H(\omega)|$  that satisfy the above equation. The filters in (b) and (c) correspond to VSB filters that retain the LSB and the USB, respectively.

It is not necessary to realize the desired VSB spectral shaping in one filter  $|H(\omega)|$ . It can be done in two stages: one filter in the transmitter  $|H(\omega)|$ , and the remaining filter at the input of the receiver. This is precisely what is done in broadcast television systems. If the two filters still do not achieve the desired vestigial characteristics, the remaining equalization can be achieved in the low-pass filter of the receiver.

# Amplitude modulation: Vestigial sideband (VSB)

**Let us learn a rule:** If the magnitude spectra of two signals are symmetric about a central frequency, the magnitude of their sum is not symmetric about that frequency.

It should be noted that signals  $g(t) \cdot \cos(\omega_0 t)$  and  $g(t) \cdot \sin(\omega_0 t)$  have band-pass magnitude spectra centered symmetrically at  $\omega_0$  (as well as at  $-\omega_0$ ). A general band-pass signal  $g(t)$  can be expressed as:

$$g_{bp}(t) = g_c(t) \cos \omega_0 t + g_s(t) \sin \omega_0 t$$

If  $g_c(t)$  and  $g_s(t)$  are both low-pass signals, each band-limited to  $B$ , then  $g_{bp}(t)$  will be a band-pass signal of band-width  $2B$ , and its spectrum will be at  $\omega_0$  (and  $-\omega_0$ ).

Although  $g_c(t) \cos(\omega_0 t)$  and  $g_s(t) \sin(\omega_0 t)$  both have magnitude spectra that are symmetric about  $\omega_0$  (and  $-\omega_0$ ), the magnitude spectrum of their sum  $g_{bp}(t)$  is not symmetric about  $\omega_0$ .

# Amplitude modulation: Vestigial sideband (VSB)

To find  $\phi_{VSB}(t)$ , we note that  $\phi_{VSB}(\omega)$  is a band-pass spectrum, **not symmetrical** about its center frequency  $\omega_c$ . Hence, it can be expressed in terms of quadrature components.

$$\phi_{VSB}(t) = m_c \cos(\omega_c t) + m_s \sin(\omega_s t)$$

We can show that  $m_c(t) = m(t)$ . When  $\phi_{VSB}(t)$  is multiplied by  $2.\cos(\omega_c t)$  and then low-pass filtered, the output is  $m(t)$ . In our case

$$2\phi_{VSB}(t) \cdot \cos \omega_c t = m_c(t) + m_c(t) \cos(2\omega_c t) + m_s(t) \sin(2\omega_c t)$$

When this signal is low pass filtered, the output is  $m_c(t)$ .

Hence, it follows that  $m_c(t) = m(t)$

To determine  $m_s(t)$ , we observe that if  $\phi_{VSB}(t)$  in Eqn.  $\phi_{VSB}(t) = m_c \cos(\omega_c t) + m_s \sin(\omega_s t)$  is multiplied by  $2.\sin(\omega_c t)$  and then low-pass filtered, the output is  $m_s(t)$ . Because \_\_\_\_\_

$$2\phi_{VSB}(t) \cdot \sin \omega_c t \leftrightarrow j[\phi_{VSB}(\omega + \omega_c) - \phi_{VSB}(\omega - \omega_c)]$$

Substituting the relation,  $\phi_{VSB}(\omega) = [M(\omega + \omega_c) + M(\omega - \omega_c)]H(\omega)$

in the above equation, we get,  $m_s(t) \leftrightarrow jM(\omega)[H(\omega + \omega_c) - H(\omega - \omega_c)]$

The use of Eqn.  $[H(\omega + \omega_c) + H(\omega - \omega_c)] = 1 \quad |\omega| \leq 2\pi B$

in the above equation yields  $m_s(t) \leftrightarrow jM(\omega)[1 - 2H(\omega - \omega_c)]$

# VSB.....

The vestigial-modulated signal  $\phi_{VSB}(t)$  is

$$\phi_{VSB}(t) = m(t) \cos \omega_c t + m_s(t) \sin \omega_c t$$

If we change the sign of  $m_s(t)$ , it amounts to turning the filter  $H(\omega)$  about  $\omega_c$ . This gives the VSB signal which retains the upper sideband. Hence,  $\phi_{VSB}(t)$  can be expressed as

$$\phi_{VSB}(t) = m(t) \cos \omega_c t \pm m_s(t) \sin \omega_c t$$

This expression is similar to that of an SSB signal with  $m_s(t)$  replacing  $m_h(t)$ .

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# VSB.....

It can be seen that VSB does the trick by partially suppressing the transmitted sideband and compensating for this with a gradual roll-off filter. Thus, VSB is a clever compromise between SSB and DSB, with advantages of both at a small cost in increased bandwidth, which is slightly larger than that of SSB (typically 25 to 30 percent larger).

It can be generated from DSB signals by using relatively simple filters with gradual roll-off characteristics. Its immunity to selective fading is comparable to that of SSB. Also, if a sufficiently large carrier  $A \cos \omega_c t$  is added, the resulting signal (VSB + C) can be demodulated by an envelope detector with relatively small distortion.

This mode (VSB + C) combines the advantages of AM, SSB, and DSB. All these properties make VSB attractive for commercial television broadcasting. The base-band video signal of television occupies an enormous bandwidth of 4.5 MHz, and a DSB signal needs a bandwidth of 9 MHz. It would seem desirable to use SSB in order to conserve the bandwidth.

Unfortunately, this creates two problems. First, the base-band video signal has sizable power in the low-frequency region, and consequently it is very difficult to suppress one sideband completely. Second, for a broadcast receiver, an envelope detector is preferred over a synchronous one in order to reduce the cost. An envelope detector gives much less distortion for VSB signals than for SSB signals.



# VSB.....

**The VSB characteristics in television signals are achieved in two steps:**

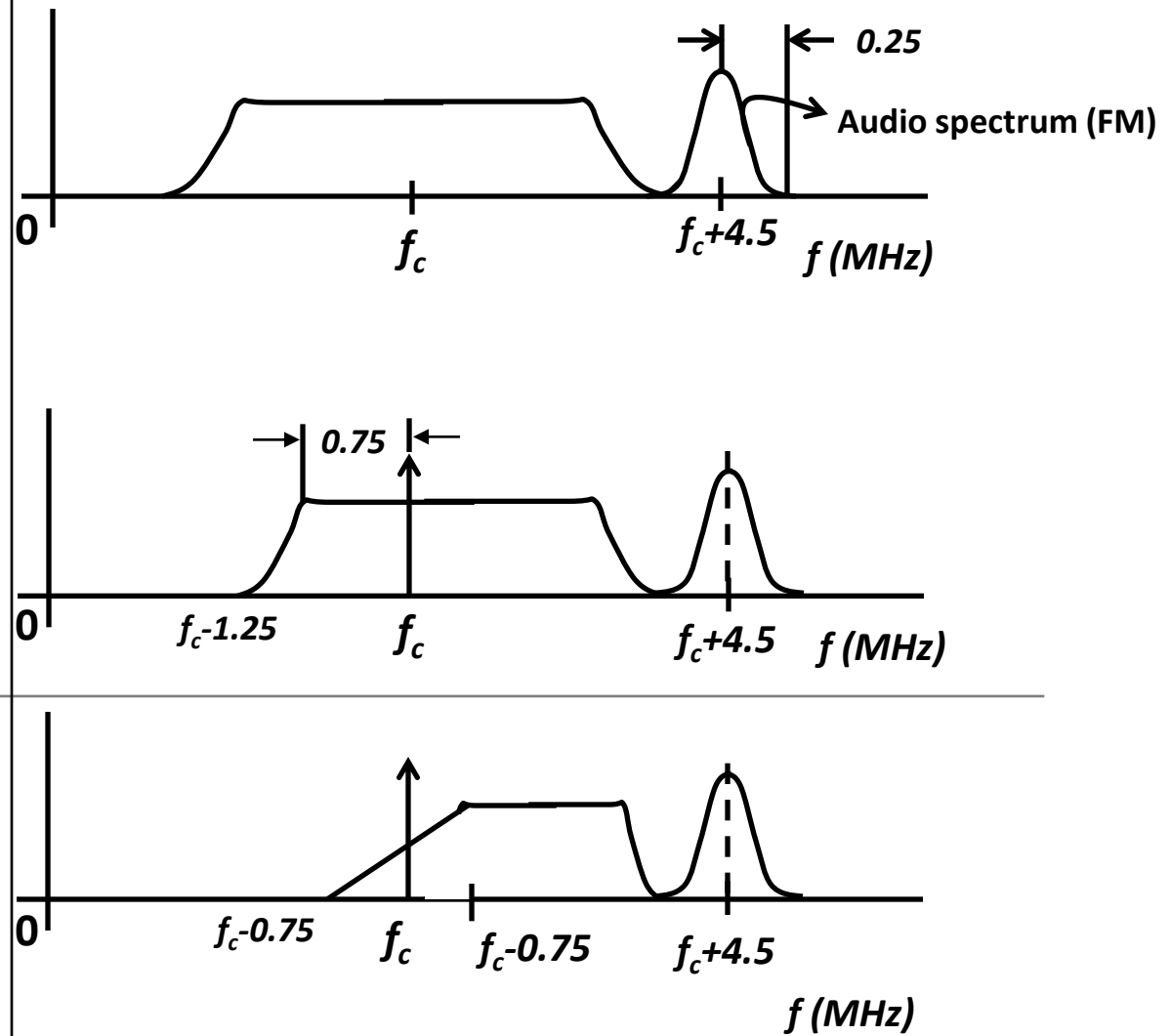
**At the transmitter, the spectrum is shaped without rigidly controlling the transition region. The receiver also has an input filter, which further shapes the incoming signal to have the desired complimentary symmetry required of a vestigial signal. The reason for the two-step shaping is that it is much easier to shape the spectrum at the receiver, where power levels are much lower.**

**The envelope detector in a VSB system does cause some distortion. Because the eye is relatively insensitive to amplitude distortion, this problem is not too serious.**

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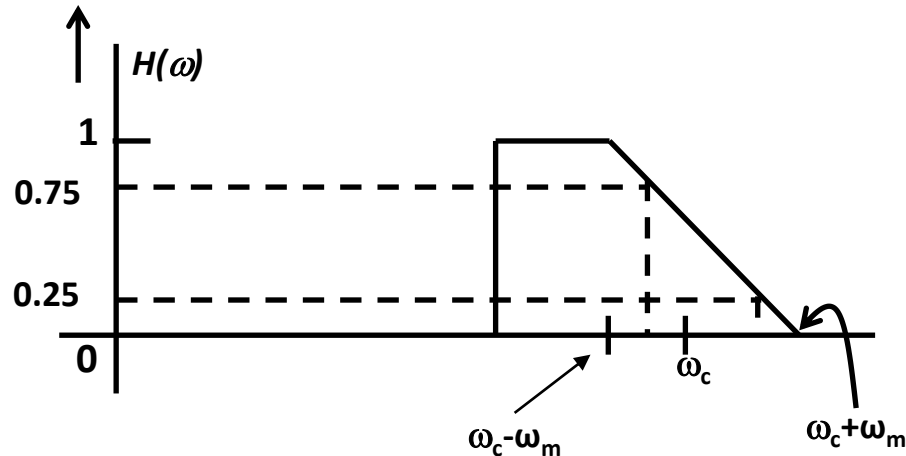
# Envelope detection of VSB + C signals

That VSB + C signals can be envelope detected may be proved by using exactly the same argument used in proving the case for SSB + C signals. This is because the modulated signals in both cases have the same form, with  $m_h(t)$  in SSB replaced by  $m_s(t)$  in VSB. We have shown that SSB + C requires a much larger carrier than DSB + C (AM) for envelope detection. Because VSB + C is an in-between case, it is expected that the added carrier in VSB will be larger than that in AM but smaller than that in SSB + C.



## Example...

A vestigial-filter transfer function  $H(\omega)$  is shown in the following figure, with  $\omega_c = 10^5$ . Find the VSB-modulated signal  $\phi_{VSB}(t)$  when  $m(t) = \cos(\omega_m t)$  ( $\omega_m = 1000$ ).



This is the case of tone modulation. The DSB signal is

$$2 \cos \omega_m (t) \cos \omega_c t = \cos(\omega_c - \omega_m)t + \cos(\omega_c + \omega_m)t$$

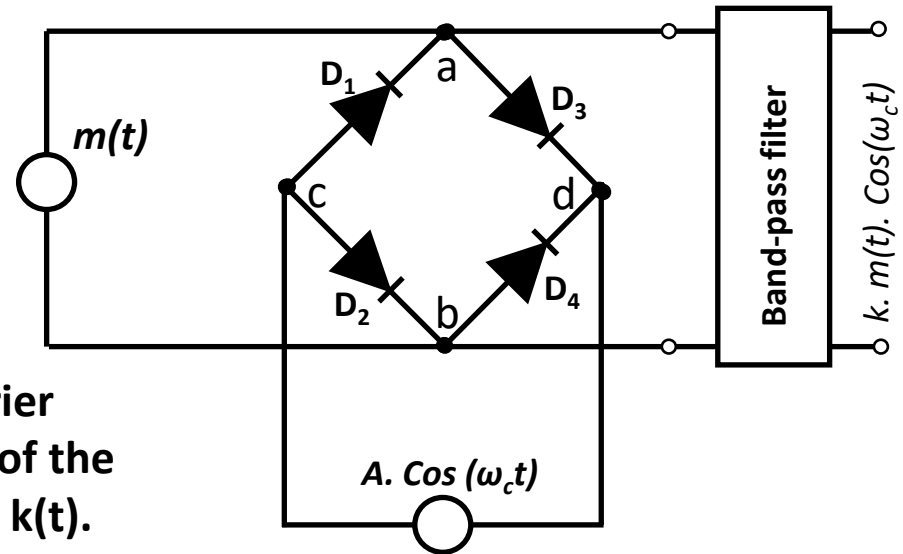
There are two sinusoids, of frequencies  $\omega_c + 1000$  and  $\omega_c - 1000$ , respectively. These sinusoids are transmitted through  $H(\omega)$  in the above figure, which has a gain of 0.75 and 0.25 at  $\omega_c - 1000$  and  $\omega_c + 1000$ , respectively. Hence  $\phi_{VSB}(t)$ , the filter output is

$$\begin{aligned} \phi_{VSB}(t) &= 0.75 \cos(\omega_c - \omega_m)t + 0.25 \cos(\omega_c + \omega_m)t \\ &= \underbrace{\cos(\omega_m t)}_{m(t)} \cos(\omega_c t) + 0.5 \underbrace{\sin(\omega_m t) \sin(\omega_c t)}_{m_s(t)} \end{aligned}$$

# Problem

Analyze the switching modulator as shown in the figure when it is used as a synchronous demodulator.

The input signal is  $m(t) \cdot \cos(\omega_c t)$ . The carrier causes the periodic switching on and off of the input signal. The output is  $m(t) \cdot \cos(\omega_c t) \cdot k(t)$ .



$$m(t) \cdot k(t) \cdot \cos \omega_c t = m(t) \cos \omega_c t \left[ \frac{1}{2} + \frac{2}{\pi} \left( \cos \omega_c t - \frac{1}{3} \cos(3\omega_c t) + \dots \right) \right]$$

$$= \frac{1}{\pi} m(t) + \text{other terms centered at } \omega_c, 2\omega_c, \dots$$

When this signal is passed through a low-pass filter, the output is the desired signal  $(1/\pi) \cdot m(t)$

# Frequency division multiplexing

- ✓ Multiplexing is a technique whereby several message signals are combined into a composite signal for transmission over a common channel.
- ✓ To transmit a number of these signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiver end.
- ✓ There are two basic multiplexing techniques: **frequency-division multiplexing (FDM)** and **time division-multiplexing (TDM)**.
- ✓ In FDM, the signals are separated in frequency, whereas in TDM, the signals are separated in time.
- ✓ Any type of modulation can be used in FDM as long as the carrier spacing is sufficient to avoid spectral overlap. However, the most widely used method of modulation is SSB modulation.

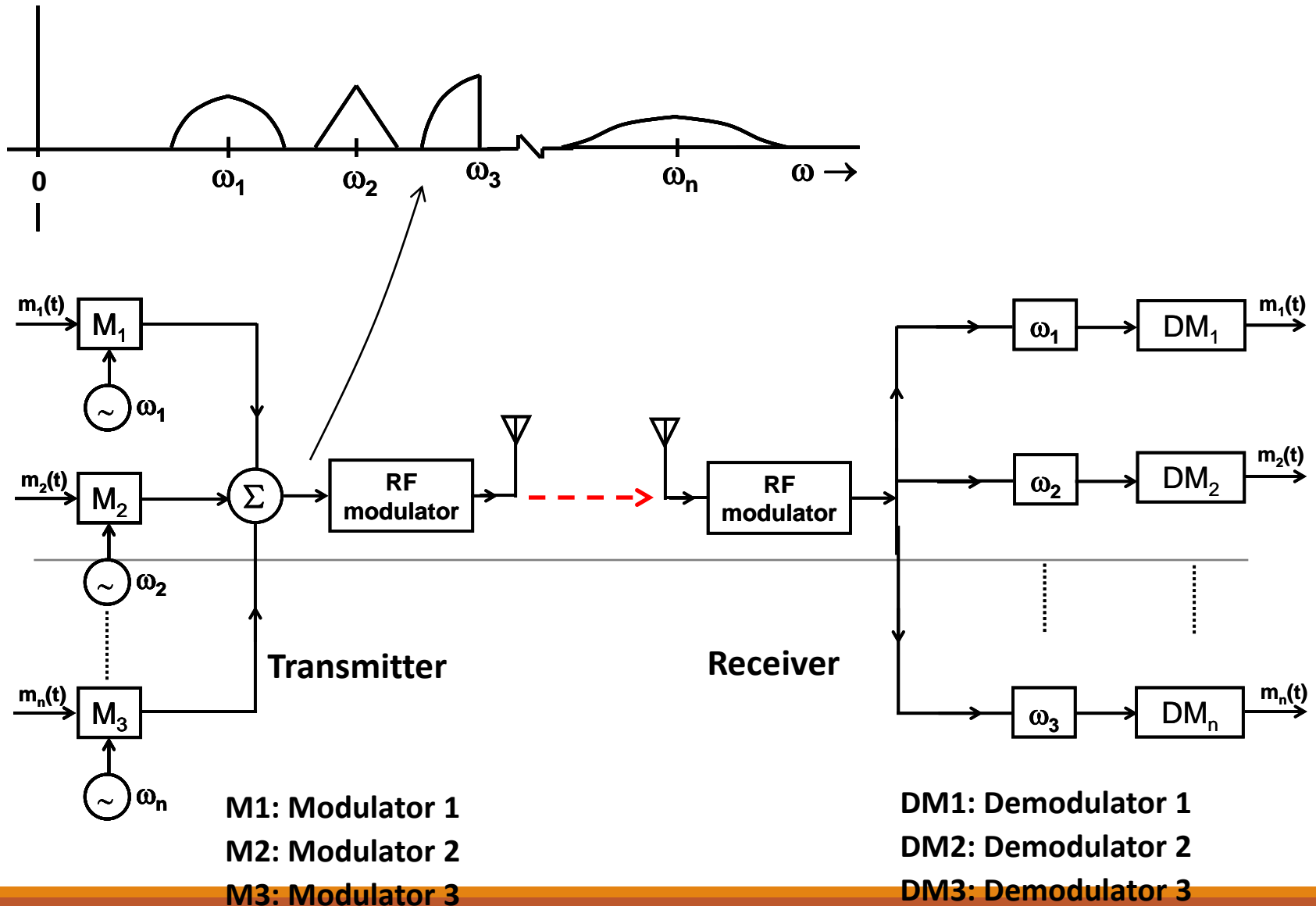
# Frequency division multiplexing

- ✓ FDM is used in telephone system, telemetry, commercial broadcast, television, and communication network.
- ✓ Commercial AM broadcast stations use carrier frequency spaced 10 kHz apart in the frequency range from 540 to 1600 kHz. This separation is not sufficient to avoid spectral overlap for AM with a reasonably high-fidelity (50 to 15 kHz) audio signal.
- ✓ Commercial FM broadcast uses carrier frequencies spaced 200 kHz apart. In a long-distance telephone system, up to 600 or more voice signals (200 to 3.2 kHz) are transmitted over a co-axial cable or microwave links by using SSB modulation with carrier frequencies spaced 4 kHz, apart.
- ✓ In practice, the composite signal formed by spacing several signals in frequency may in turn, be modulated by using another carrier frequency. In this case, the first carrier frequencies are called sub-carriers.

# Frequency Division Multiplexing (FDM)

- Each signal is modulated by a **different carrier frequency**. The various carriers are adequately separated to avoid overlap (or interference) between the spectra of various modulated signals. These carriers are referred to as sub-carriers.
- Each signal may use a different kind of modulation (for example, DSB-SC, AM, SSB, VSB, or even FM or PM). The modulated-signal spectra may be separated by a small guard band to avoid interference and facilitate signal separation at the receiver.
- When all of the modulated spectra are added, we shall have a composite signal that may be considered as a base-band signal to further modulate a high-frequency (radio frequency, or RF) carrier for the purpose of transmission.
- At the receiver, the incoming signal is first demodulated by the RF carrier to retrieve the composite base-band, which is then band-pass filtered to separate each modulated signal. Then each modulated signal is individually demodulated by an appropriate sub-carrier to obtain all the basic base-band signals.

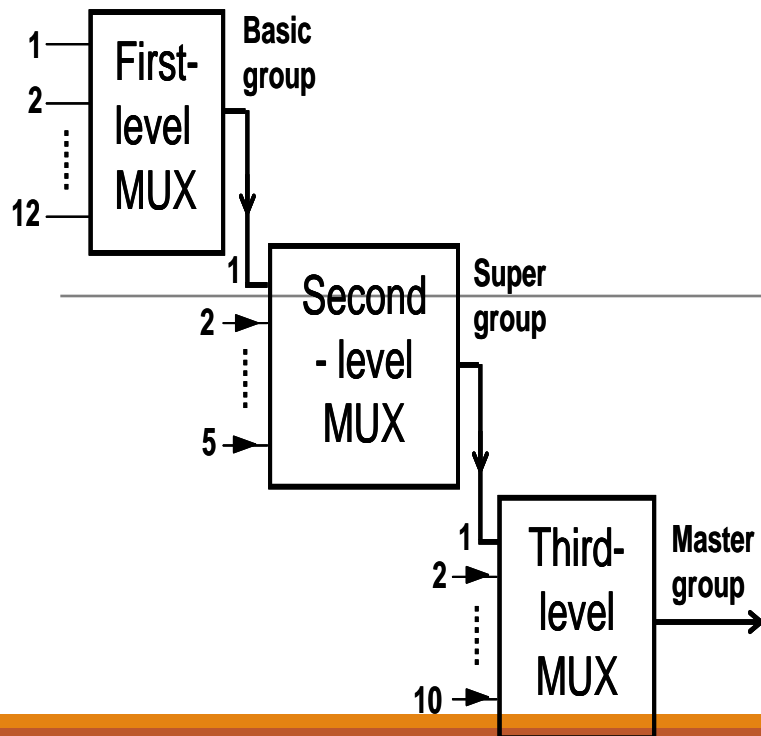
# Frequency division multiplexing (FDM)





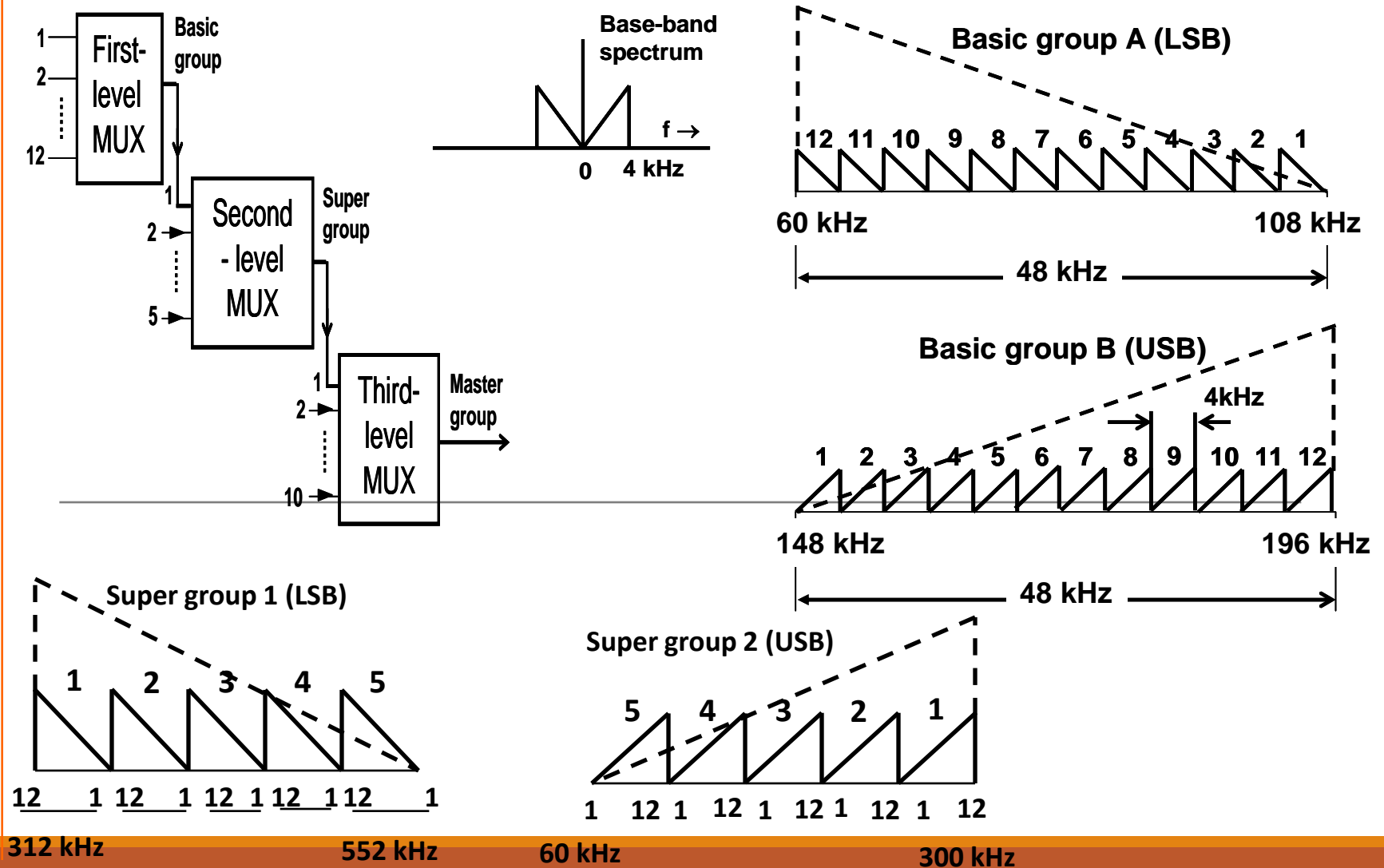
# FDM: telephone channel multiplexing

Almost all long-haul telephone channels were multiplexed by FDM using SSB signals. This multiplexing technique, standardized by the CCITT, provides considerable flexibility in branching, dropping off, or inserting blocks of channels at points. The typical arrangement in North American FDM telephone hierarchy is shown below.

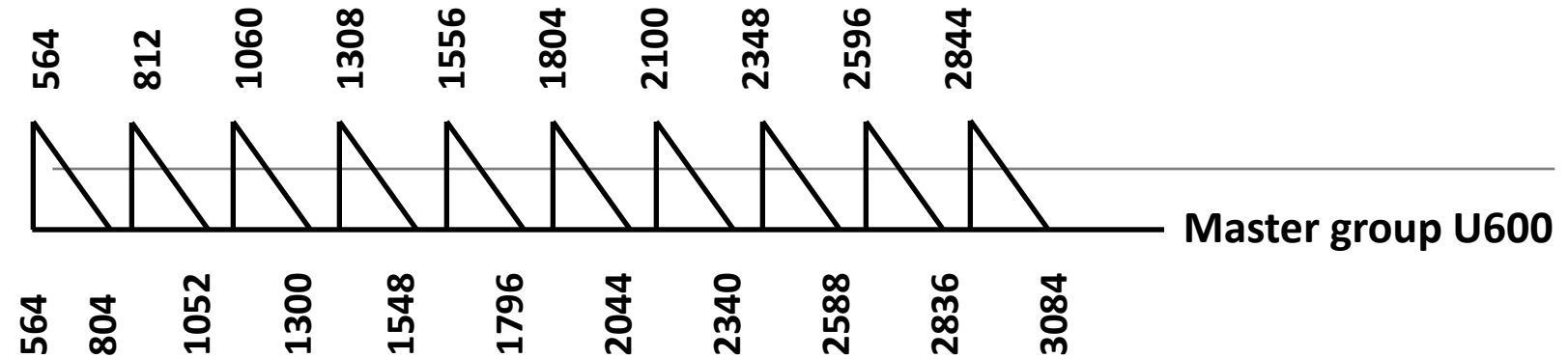
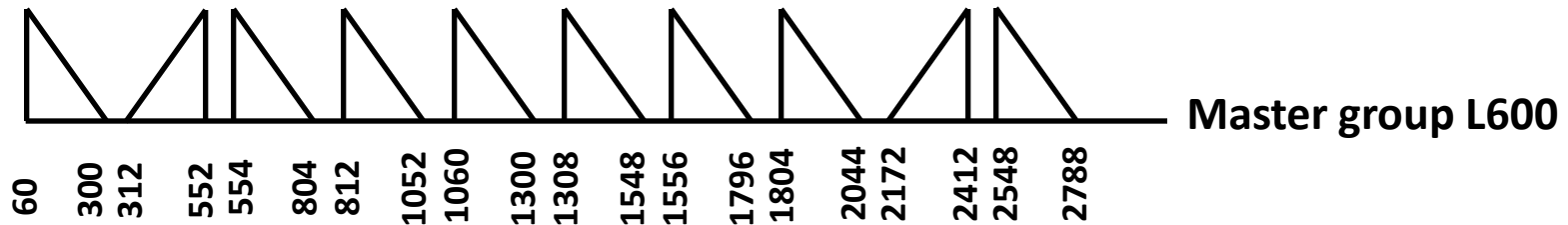


- A basic group consists of 12 FDM SSB voice channels, each of bandwidth 4 kHz (first-level multiplexing). A basic group uses LSB spectra and occupies a band 60 to 108 kHz.
- A basic super-group of 60 channels is formed by multiplexing five basic groups, and it occupies a band 312 to 552 kHz.
- A basic master-group of 600 channels is formed by multiplexing 10 super-groups. There are two standard master-group configurations: L600 and U600.

# Frequency Division Multiplexing



# Master group, 600 channels



Modern broadband transmission systems can transmit even larger groupings than master groups.