

Simultaneity and order of Events

'*Simultaneity*' means 'occurring, operating, or done at the same time'. On the other hand the '*Order of Events*' is the occurrence of events in a particular time sequence.

Let us discuss the meaning of the statements such as "Events A and B occurred at the same time." When we say that a train arrives at 7 o'clock this means that the exact pointing of the clock hand to 7 and the arrival of the train at the clock were simultaneous. We certainly shall not have a universal time scale if different inertial observers disagree as to whether two events are simultaneous. Let us first try to set up an unambiguous time scale in a single frame of reference; then we can set up time scales in the same way in all inertial frames and compare what different observers have to say about the sequence of two events, A and B .

Suppose that the events occur at the same place in one particular frame of reference. We can have a clock at that place which registers the time of occurrence of each event. If the reading is the same for each event, we can logically regard the events as simultaneous.

Let us now move to the situation where two events occur at different locations. Imagine now that there is a clock at the positions of each event—the clock at A being of the same nature as that at B , of course. These clocks can record the time of occurrence of the events. But, before we can compare their readings, we must be sure that they are synchronized. But some "obvious" methods of synchronizing clocks turn out to be erroneous. For example, we can set the two clocks so that they always read the same time as seen by observer A . This means that whenever A looks at the B clock it reads the same to him as his clock. The defect here is that if observer B uses the same criterion (that is, that the clocks are synchronized if they always read the same time to him), he will find that the clocks are not synchronized if A says that they are. For this method neglects the fact that it takes time for light to travel from B to A and vice versa. If the distance between the clocks is L , one observer will see the other clock lag his by $2L/c$ when the other observer claims that they are synchronous. We certainly cannot have observers in the same reference frame disagree on whether clocks are synchronized or not, so we reject this method. An apparent way out of this difficulty is simply to set the two clocks to read the same time and then move them to the positions where the events occur. (In principle, we need clocks everywhere in our reference frame to record the time of occurrence of events, but once we know how to synchronize two docks we can, one by one, synchronize all the clocks.) The difficulty here is that we do not know ahead of time, and therefore cannot assume, that the motion of the clocks (which may have different velocities, accelerations, and path lengths in being moved into position) will not affect their readings or time-keeping ability. Even in classical physics, the motion can affect the rate at which clocks run. Hence, the logical thing to do is to put our clocks into position and synchronize them using signals. As signals require a finite time to travel some distance, the time increasing with the distance travelled. The best signal to choose would be one whose speed depends on a few factors as possible. We choose electromagnetic waves because they do not require a material medium for transmission and their speed in the vacuum does not depend on their wavelength, amplitude, or direction of propagation. Furthermore, their propagation speed is the highest known and their speed is the same for all inertial observers.

Now we must account for the finite time of transmission of the signal and our clocks can be synchronized. To do this let us imagine an observer with a light source that can be turned on and off (e.g., a flashbulb) at each clock, A and B . Let the measured distance between the

clocks (and observers) be L . The agreed-upon procedure for synchronization then is that A will turn on his light source when his clock reads $t = 0$ and observer B will set his clock to $t = L/c$ the instant he receives the signal. This accounts for the transmission time and consistently synchronizes the clocks. For example, if B turns on his light source at t by his clock, the signal will arrive at A at a time $t + L/c$, which is just what A 's clock will read when A receives the signal. A method equivalent to the above is, to put a light source at the exact midpoint of the straight line connecting A and B and inform each observer to put his clock at $t = 0$ when the turned-on light signal reaches him. The light will take an equal amount of time to reach A and B from the midpoint so that this procedure does indeed synchronize the clocks.

Now that we have a procedure for synchronizing clocks in one reference frame, we can judge the time order of events in that frame. The time of an event is measured by the dock whose location coincides with that of the event. Events occurring at two different places in that frame must be called simultaneous when the docks at the respective places record the same time for them. Suppose that one inertial observer does find that two separated events are simultaneous. Let us check whether these same events be measured as simultaneous by an observer on another inertial frame which is moving with speed v with respect to the first.

Let there be two inertial reference frames S' and S having a relative velocity. Each frame has its metre sticks and synchronized clocks. The observers note that two lightning bolts strike each, hitting and leaving permanent marks in the frames. (*The essential point is to have light sources that leave marks. Exploding sticks of dynamite would do as well.*) Assume that afterwards, by measurements, each inertial observer finds that he was located exactly at the midpoint of the marks which were left on his reference frame. In Fig. 1a, these marks are left at A, B on the S -frame and at A', B' on the S' frame, the observers being at O and O' .

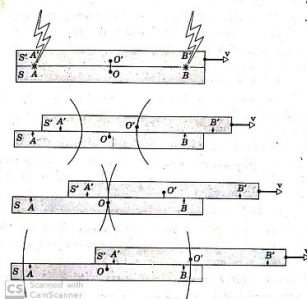
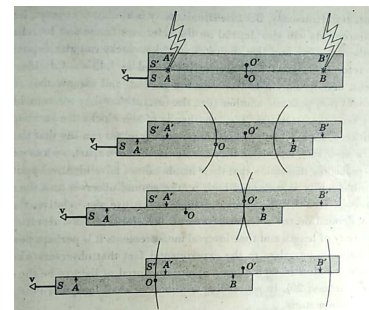


Fig. 1. The point of view of the S -frame, the S' -frame moving to the right. A light wave leaves A, A' and B, B' in (a). Successive drawings correspond to the assumption that event AA' and event BB' are simultaneous in the S -frame. In (b) one wavefront reaches O' . In (c) both wavefronts reach O . In (d) the other wavefront reaches O' .

Fig. 2. The point of view of the S' -frame, the S -frame moving to the left. A light wave leaves A, A' and B, B' in (a). Successive drawings correspond to the assumption that the event AA' and the event BB' are simultaneous in S' -frame. In (b) one wavefront reaches O . In (c) both wavefronts reach O' . In (d) the other wave-front reaches O .



Because each observer knows he was at the midpoint of the mark left by these events, he will conclude that they were simultaneous if the light signals from them arrive simultaneously at his clock. If, on the other hand, one signal arrives before the other, he will conclude that one event preceded the other. That is the observer can determine the time order of the events. Since each observer has a synchronized set of clocks, he can conclude either that the clocks at

the marks read the same time when the marks were made (simultaneous case) or that they read different times (non-simultaneous case).

In Figs. 1b to 1d we take the point of view of the S -observer and see the S' -frame moving, say, to the right. At the instant the lightning struck at A and A' , these two points coincide, and at the instant the lightning struck at B and B' those two points coincide. The S -observer found these two events to occur at the same instant, so that at that instant O and O' must coincide also for him. However, the light signals from the events take a finite time to reach O and during this time O' travels to the right (Figs. 1b to 1d). Hence, the signal from event BB' arrives at O' (Fig. 1b) before it gets to O (Fig. 1c), whereas the signal from event AA' arrives at O (Fig. 1c) before it gets to O' (Fig. 1d). Consistent with our starting assumption, the S -observer finds the events to be simultaneous (both signals arrive at O at the same instant). The S' -observer, however, finds that event BB' precedes event AA' in time; they are not simultaneous to him. Therefore, two separated events which are simultaneous with respect to one frame of reference are not necessarily simultaneous with respect to another frame.

Now we could have supposed, just as well, that the lightning bolts struck so that the S' -observer found them to be simultaneous. In that case the light signals reach O' simultaneously, rather than O . We show this in Fig. 2 where now we take the point of view of S' . The S -frame moves to the left relative to the S' -observer. But, in this case, the signals do not reach O simultaneously; the signal from event AA' reaches O before that from event BB' . Here the S' -observer finds the events to be simultaneous but the S -observer finds that event AA' precedes event BB' .

Hence, neither frame is preferred nor the situation is perfectly reciprocal. Simultaneity is genuinely a relative concept, not an absolute one. Neither observer can assert absolutely that he is at rest. Instead, each observer correctly states only that the other one is moving relative to him and that the signals travel with finite speed c relative to him. It should be clear that if we had an infinitely fast signal, then simultaneity would be an absolute concept; for the frames would not move at all relative to one another in the (zero) time it would take the signal to reach the observers. Some other conclusions suggest themselves from the relativity of simultaneity. To measure the length of an object means to locate its end points simultaneously. Because simultaneity is a relative concept, length measurements will also depend on the reference frame and be relative. Furthermore, we find that the rates at which clocks run also depend on the reference frame. This can be illustrated as follows. Consider two clocks, one on a train and one on the ground, and assume that at the moment they pass one another (i.e., the instant that they are coincident) they read the same time (i.e., the hands of the clocks are in identical positions). Now, if the clocks continue to agree, we can say that they go at the same rate. But, when they are a great distance apart, we know from the preceding discussion that their hands cannot have identical positions simultaneously as measured both by the ground observer and the train observer. Hence, time interval measurements are also relative, that is, they depend on the reference frame of the observer. As a result of the relativity of length and time interval measurements it is perhaps possible to reconcile ourselves to the experimental fact that observers who are moving relative to each other measure the speed of light to be the same.