Michelson-Morley experiment

Maxwell's equations of electromagnetism, from which we deduce the electromagnetic wave equation contain the constant$c = 1/\sqrt{\mu_0\varepsilon_0}$, which is identified as the velocity of propagation of a plane wave in vacuum. But such a velocity cannot be the same for observers in different inertial frames, according to the Galilean transformations, so that electromagnetic effects will probably not be the same for different inertial observers. In fact, Maxwell's equations are not preserved in form by the Galilean transformations, although Newton's laws are. But if we accept both the Galilean transformations and Maxwell's equations as basically correct, then it automatically follows that there exists a unique privileged frame of reference (the "ether" frame) in which Maxwell's equations are valid and in which light is propagated at a speed$c = 1/\sqrt{\mu_0\varepsilon_0}$.

The obvious experiment would be one in which we can measure the speed of light in a variety of inertial systems, noting whether the measured speed is different in different systems, and if so, noting especially whether there is evidence for a single unique system–the "ether" frame–in which the speed of light is$c$, the value predicted from electromagnetic theory. A. A. Michelson in 1881 and Michelson and E. W. Morley in 1887 carried out such an experiment.Let us now describe the Michelson-Morley experiment. The Michelson interferometer (Fig. 1) is fixed on the earth.

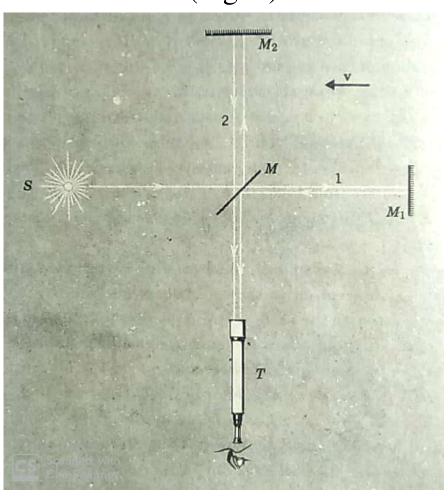


Fig.1:*A simplified version of the Michelson interferometer showing how the beam from the source S is split into two beams by the partially silvered mirror M. The beams are reflected by mirrors 1 and 2, returning to the partially silvered mirror. The beams are then transmitted to the telescope T where they interfere, giving rise to a fringe pattern. In this figure, v is the velocity of the ether with respect to the interferometer.*

If we imagine the "ether" to be fixed with respect to the sun, then the earth (and interferometer) moves through the ether at a speed of 30 km/sec, in different directions in different seasons (Fig. 2). For the moment, neglect the earth's spinning motion.

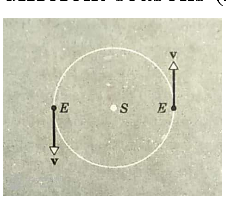


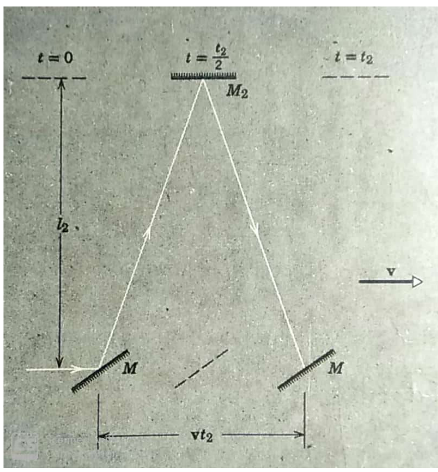
Fig. 2.*The earth E moves at an orbital speed of 30 km/sec along itsnearly circular orbit about the sun S,reversing the direction of its velocityevery six months.*

The beam of light (plane waves, or parallel rays) from the laboratory source S (fixed with respect to the instrument) is split by the partially silvered mirror M into two coherent beams, beam 1 being transmitted through M and beam 2 being reflected off of M. Beam 1 is reflected back to M by mirror M_1 and beam 2 by mirror M_2 . Then the returning beam 1 is partially reflected and the returning beam 2 is partially transmitted by M back to a telescope at T where they interfere. The interference is constructive or destructive depending on the phase difference of the beams. The partially silvered mirror surface M is inclined at 45^0 to the beam directions. If M_1 and M_2 are very nearly (but not quite) at right angles, we shall observe a fringe system in the telescope consisting of nearly parallel lines.

The time for beam 1 to travel from M to M_1 and back is

$$t_1 = \frac{l_1}{c - v} + \frac{l_2}{c + v} = l_1\left(\frac{2c}{c^2 - v^2}\right) = \frac{2l_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

for the light, whose speed is c in the ether, has an "upstream" speed of $c - v$ with respect to the apparatus and a "downstream" speed of $c + v$.



The cross-stream path of beam 2. The mirrors movethrough the "ether" at a speed v, the light moving through the"ether" at speed c. Reflection from the moving mirror automatically gives the cross-stream path. In this figure, v is the velocity ofthe interferometer with respect to the "ether."

The path of beam 2, travelling from M to M_2 and back, is a cross-stream path through the ether enabling the beam to return to the (advancing) mirror M. The transit time is given by

$$2\left[{l_2}^2 + \left(\frac{vt_2}{2}\right)^2\right] = ct_2$$

$$t_2 = \frac{2l_2}{\sqrt{c^2 - v^2}} = \frac{2l_2}{c}\left(\frac{1}{\sqrt{1 - v^2/c^2}}\right)$$

The calculation of t_2 is made in the ether frame, that of t_1 in the frame of the apparatus. Because time is an absolute in classical physics, this is perfectly acceptable classically. Note that both effects are second-order ones ($v^2/c^2 \approx 10^{-8}$) and are in the same direction (they increase the transit time over the case $v = 0$).The difference in transit times is

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{l_2}{\sqrt{1 - v^2/c^2}} - \frac{l_1}{1 - v^2/c^2}\right)$$

Suppose that the instrument is rotated through 90^0, thereby making $l_1$the cross-stream length and l_2 the downstream length. If the corresponding times are now designated by primes, the same analysis as above gives the transit-time difference as

$$\Delta t' = t_2{}' - t_1{}' = \frac{2}{c}\left(\frac{l_2}{1 - v^2/c^2} - \frac{l_1}{\sqrt{1 - v^2/c^2}}\right)$$

Hence, the rotation changes the differences by

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_2 + l_1}{1 - v^2/c^2} - \frac{l_2 + l_1}{\sqrt{1 - v^2/c^2}}\right)$$

Using the binomial expansion and dropping terms higher than the second-order, we find

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left(1 + \frac{v^2}{c^2} - 1 - \frac{1}{2}\frac{v^2}{c^2}\right) = \left(\frac{l_1 + l_2}{c}\right)\frac{v^2}{c^2}$$

Therefore, the rotation should cause a shift in the fringe pattern, since it changes the phase relationship between beams 1 and 2.Let ΔN represent the number of fringes moving past the crosshairs as the pattern shifts. Then, if light of wave-length λ is used, so that the period of one vibration is $T = 1/\upsilon = \lambda/c$

$$\Delta N = \frac{\Delta t' - \Delta t}{T} = \left(\frac{l_1 + l_2}{cT}\right)\frac{v^2}{c^2} = \left(\frac{l_1 + l_2}{\lambda}\right)\frac{v^2}{c^2}$$

Michelson and Morley were able to obtain an optical path length,$l_1 + l_2$of about 22 m. In their experiment the arms were of (nearly) equal length, that is,$l_1 = l_2 = l$so that $\Delta N = (2l/\lambda)(v^2/c^2)$. If we choose $\lambda = 5.5 \times 10^{-7}\, m$ and $v/c = 10^{-4}$, we obtain

$$\Delta N = \frac{22m}{5.5 \times 10^{-7}\, m} 10^{-8} = 0.4$$

or a shift of four-tenths a fringe! But no such expected fringe shift was observed. Indeed, the experimental conclusion was that there was no fringe shift at all.The null result seems to rule out an ether (absolute) frame.

The postulate of Special theory of Relativity

1.The laws of physics are the same in all inertial systems. No preferred inertial system exists. (The Principle of Relativity.)

2. The speed of light in free space has the same value c in all inertial systems. (The Principle of the Constancy of the Speed of Light.)