SOLUTION OF DIFFERENTIAL EQUATION WITH ODEINT

Differential equations can be solved in Python with the Scipy.integrate package using function **ODEINT**. ODEINT requires three inputs:

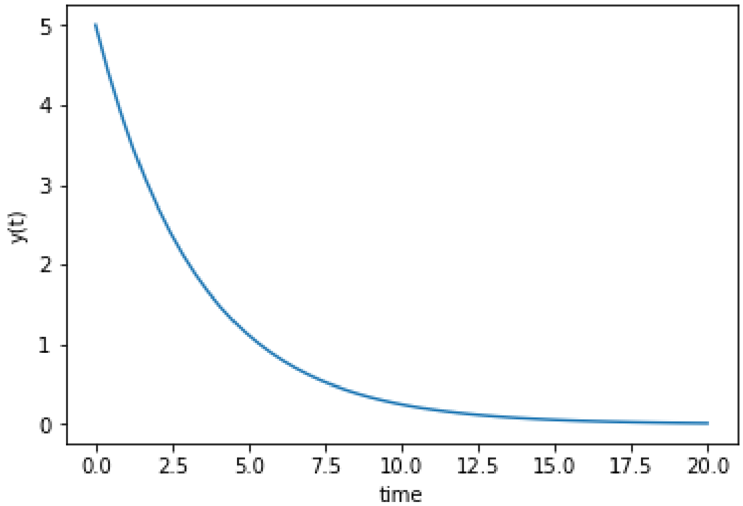
```
y = odeint(model, y0, t)
```

1. **model**: Function name that returns derivative values at requested y and t values as dydt = model(y,t)
2. **y0**: Initial conditions of the differential states
3. **t**: Time points at which the solution should be reported

Ex.1 Solve an ordinary differential equation $\frac{dy(t)}{dt} = ky(t)$ for y(t), with initial condition y_0=3 and parameter k=0.4

The Python code first imports the needed Numpy, Scipy, and Matplotlib packages. The model, initial conditions, and time points are defined as inputs to *ODEINT* to numerically calculate *y(t)*.

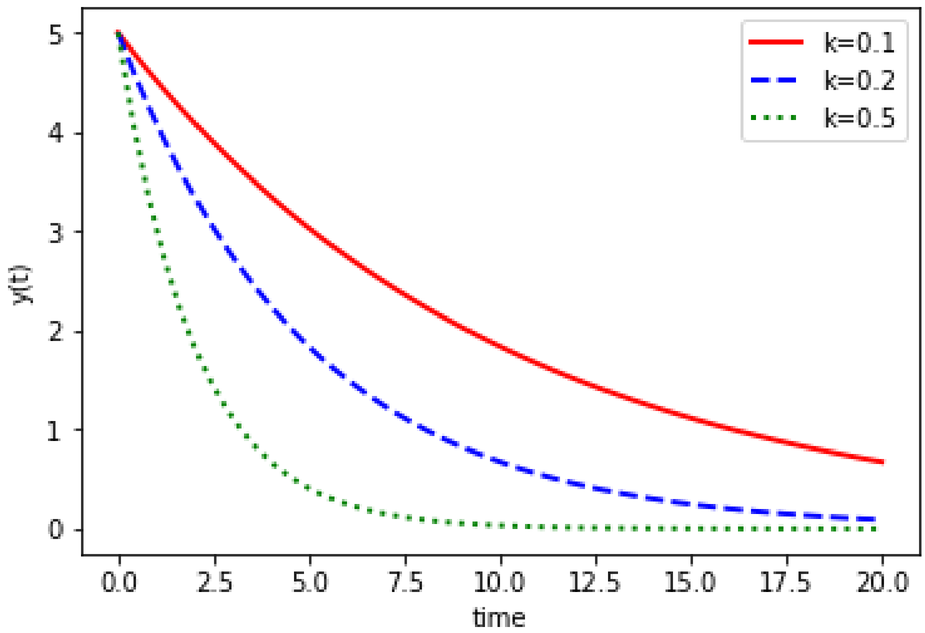
```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

# function that returns dy/dt
def model(y,t):
    k = 0.4
dydt = -k * y
    return dydt

# initial condition
y0 = 3

# time points
t = np.linspace(0,20)

# solve ODE
y = odeint(model,y0,t)

# plot results
plt.plot(t,y)
plt.xlabel('time')
plt.ylabel('y(t)')
plt.show()
```



Ex.2 **The argument *k* is now an input to the *model* function by including an addition argument.**

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt

# function that returns dy/dt
def model(y,t,k):
dydt = -k * y
    return dydt

# initial condition
y0 = 5

# time points
t = np.linspace(0,20)

# solve ODEs
k = 0.1
y1 = odeint(model,y0,t,args=(k,))
k = 0.2
y2 = odeint(model,y0,t,args=(k,))
k = 0.5
y3 = odeint(model,y0,t,args=(k,))

# plot results
plt.plot(t,y1,'r-',linewidth=2,label='k=0.1')
plt.plot(t,y2,'b--',linewidth=2,label='k=0.2')
plt.plot(t,y3,'g:',linewidth=2,label='k=0.5')
plt.xlabel('time')
plt.ylabel('y(t)')
```

```
plt.legend()
plt.show()
```



Exercises

Find a numerical solution to the following differential equations with the associated initial conditions. Expand the requested time horizon until the solution reaches a steady state. Show a plot of the states (*x(t)* and/or *y(t)*). Report the final value of each state as t→∞t→∞.

Problem 1 **solve for y(t)** $\frac{dy(t)}{dt} = y(t) + 1$ with initial condition y(0)=0

Problem 2 **solve for y(t)** $5\frac{dy(t)}{dt} = y(t) + u(t)$ with initial condition y(0)=1
u steps from *0* to *2* at t=10

Problem 3 **Solve for x(t) and y(t) and show that the solutions are equivalent**.

$\frac{dx(t)}{dt} = 3exp(-t)$, $\frac{dy(t)}{dt} = 3 - y(t)$with initial condition x(0)=0 and y(0)=0
